

## Open Problems from 2007 Fall Workshop in Computational Geometry

### Union of Translates of a Convex Set

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Let  $C \subset \mathbb{R}^d$ ,  $d > 1$ , be a convex set. Consider the union  $U := \bigcup_{i=1, \dots, k} C + t_i$  of  $k$  translates of  $C$ .

Conjecture: The number of connected components of the complement of  $U$  is at most  $c_d k^{d-1}$ , with the constant  $c_d$  independent of choice of  $C$ .

Known:

- Holds in the plane. A consequence of the fact that translates behave like pseudodisks.

- Known in all dimensions for simple shapes: unit balls, axis-aligned unit cubes; in fact, the exponent is  $\lfloor d/2 \rfloor$  in that case.

- The number is  $O(k^d)$  for any shape  $C$ . In fact, the maximum number of connected components of the complement of the union of  $k$  arbitrary convex shapes (unrelated to each other) is  $\Theta(k^d)$ ; the exact expression is known, and the class of objects for which it is achieved is known.

More ambitious generalizations:

- Homothets, not just translates
- Maybe the right exponent is  $\lfloor d/2 \rfloor$  for translates, and  $\lceil d/2 \rceil$  for homothets?

Simplest open problem:

- Obtain a subcubic (ideally, quadratic) upper bound in 3D, when, say,  $C$  is a convex polytope with any number of faces. The constant of proportionality should be independent of  $C$ .

### Iterated Tangent

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For  $n$  a nonnegative integer, let  $b(0) = 1$ ,  $b(n) = \tan(b(n-1))$ , and  $a(n) = \lfloor b(n) \rfloor$ . Conjecture: Every integer appears in the sequence  $a(n)$ .

### Pizza Wrapping

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Given a slice of pizza and silver foil dispensed from a foil dispenser in one rectangular sheet, what

is the minimum amount of silver foil needed to completely cover the front and back of the pizza slice given that we are able to fold but not tear the silver foil? Assume the pizza slice is an eighth of a circular disk, of radius  $r$  inches, and infinitely thin, and that the dispenser dispenses rectangular sheets of side length  $\ell$  inches. For a typical pizza,  $r = 8$ , and for a typical silver foil dispenser,  $\ell = 12$ .

### Evel Knievel Problem

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Given an arrangement of  $n$  lines in the plane (or more generally a polygonal subdivision of  $\mathbb{R}^2$ ). A polygonal path  $\pi$  from  $s$  to  $t$  is *orthogonal* with respect to the subdivision if (a) there is at most one vertex (turn point) of  $\pi$  per face of the subdivision, and (b) the path  $\pi$  crosses each edge of the subdivision (line in the arrangement) orthogonally. We conjecture that for any  $s$  and  $t$  interior to faces of the subdivision, there is an orthogonal path from  $s$  to  $t$ . (In fact, it may be that there is an orthogonal path having no turn point in the face containing  $s$  (or  $t$ ).

The mental picture given for this problem is that the lines of the arrangement are “canyons” that a motorcycle is to jump, by crossing each orthogonally (maximizing the chance of clearing the canyon).

### Dark Points Among Disks

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Let  $D$  be a disk of radius  $r$  in the plane. We say that a set  $S$  of unit disks within  $D$  is a *maximal packing* if the unit disks are pairwise-disjoint and the set is maximal (it is not possible to add another disk to the set  $S$  while maintaining the packing property). We say that a point  $p$  is *dark* within the “forest” defined by  $S$  if any ray with apex  $p$  intersects some disk of  $S$  (i.e., a person standing at  $p$  cannot “see” out of the forest). We conjecture that for large enough radius  $r$  of  $D$ , there exists a dark point  $p$  for any maximal packing of unit disks within  $D$ .

**Anisotropic Bricks****Joseph O'Rourke****Smith College****orourke@cs.smith.edu**

A *brick* is a  $k \times 1 \times 1$  rectangular box. Let a rectangular box  $B$  be packed with bricks, so that there are no gaps. View the boundaries between bricks as filled with (infinitesimally thin) cement. We now define a measure of the disorder of the brick packing.

Let  $P$  be a lattice coordinate plane, a plane parallel to a side of box  $B$ , aligned along an integer lattice. Let  $S(P)$  be the regularization of the set of cement points in  $P \cap B$ . The regularization is achieved by taking the closure of the interior, which removes isolated points and one-dimensional "spurs," leaving a two-dimensional set. The combinatorial complexity of  $S(P)$  is the number of (maximal) segments comprising its boundary. The *disorder* of the packing is the sum of the combinatorial complexities of  $S(P)$  over all  $P$ .

For example, let  $k = 2$ , and consider the simple stacking of one brick on top of another. Then, when  $P$  does not align with a crack between bricks,  $P \cap B$  is a one-dimensional grid of cement points, which regularization reduces to the empty set. When  $P$  does align with a crack,  $P \cap B$  is a rectangle of cement, with complexity 4. This packing has very low disorder.

On the other hand, if one staggers the bricks as in a standard brick wall, a  $P$  that cuts transversely through the wall has complexity  $4(n/2)$  for height  $n$ , because  $P \cap B$  alternates between unit squares of cement and brick-interior.

The problem is to maximize the disorder, attempting to achieve the staggered brick effect in all directions, throughout  $B$ .

**Inevitability of a Continental Divide****Richard Pollack****Courant Institute****pollack@cims.nyu.edu**

Given an island with arbitrary smooth relief profile, is there necessarily a curve extending from one point on the boundary to another, such that if sufficient rain fell uniformly over the island, any raindrop falling on one side or the other of the curve would follow a path from that point to the sea always remaining on the same side of the curve?