Fundamental limits to magnetic field sensitivity of fluxgate magnetic field sensors

R.H. Koch, J.G. Deak, and G. Grinstein
IBM Thomas J. Watson Research Center
Yorktown Heights, NY 10598
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In this paper we determine the theoretical limit of the magnetic field sensitivity of the fluxgate magnetometer. In order to do so, we have developed a model for the white noise of a fluxgate based on the fundamental dynamics of the magnetic material forming the fluxgate core. Solving this model we predict that the white noise of a physically realizable fluxgate with a volume of $2 \times 10^{-8}\ m^3$ is less than $100 fT/\sqrt{Hz}$. The white noise varies with the lossy susceptibility of the core and inversely with the volume. We also compare the measured white noise of a thin film fluxgate with the predictions of our model and find that the measured and predicted noise agree reasonably well.

In this paper we examine the question; What is the theoretical limit of the magnetic field sensitivity of the fluxgate magnetometer. A fluxgate is a magnetometer that uses a ferromagnetic core, usually operating at room temperature, which can be used to measure magnetic fields with a sensitivity of aboug 1 to 10 $pT/\sqrt{Hz}$ at 1 Hz [1]. Most other types of magnetic field sensors, including the Superconducting QUantum Interference Device (SQUID), the induction coil, and MagnetoResistance (MR) sensor have simple and precise models for the best obtainable field sensitivity. However, the complexities associated with the operation of a fluxgate as a nonlinear device, and the observation that the measured noise is usually 1/f noise, makes predicting the noise complicated. A complete model for the noise of a fluxgate should include predictions for the white, or frequency-independent, noise from the core as well as the low frequency or 1/f noise. There have been many papers discussing the low frequency noise of the fluxgate magnetometer [1] [2]. However for the white noise generated from the core, as far as we know, there haven't been any model predictions.

In this paper we will model the white noise of the fluxgate in two ways. We start with a simple model based on the equilibrium magnetic fluctuations of the fluxgate core. This method makes a prediction for the noise that does not include effects of the applied drive magnetic field which alternately saturates the core in either direction. A more complete treatment, which includes the influence of the drive field, is to use the Landau-Lifshitz-Gilbert (LLG) equation to predict the dynamical behavior of the magnetization with the applied alternating magnetic field. We solve the LLG equation in the presence of a Langevin noise term which represents the equilibrium magnetic fluctuations of the core. In order to test our model we have also fabricated thin film fluxgates from permalloy films and used these devices to measure the white noise [3]. We find reasonable agreement between the predicted noise of the model and the measured noise of our thin film fluxgates.

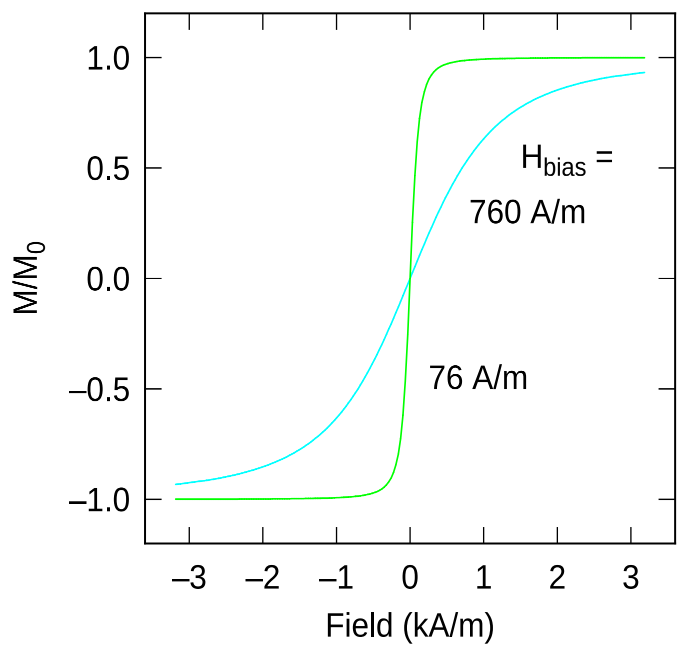

FIG. 1. Magnetization vs. field (M vs. H) curve for simulated fluxgate with hard axis bias fields of 760 and 76 A/m. T = 300 K.

The fluxgate is modeled as a core of magnetic material having a magnetization vs. field of a single magnetic domain. The magnetic core is alternately driven to the saturated state in either direction by current through a drive coil. The output of the fluxgate is a voltage measured across a sense coil. The output is multiplied by the second harmonic of the drive current and the averaged value of the product is proportional to the DC field applied to the fluxgate. In this model we approximate the core as a single magnetic domain. This approximation is valid in our experiment because when we operate the fluxgate, we apply a uniform hard axis bias field, H_B, in the plane of the film perpendicular to the drive field [3]. This aligns the magnetic domains of the core in that direction. We have also made a fluxgate that uses a core geometry of a tube to insure the core is a single magnetic domain [4]. The process of alternately saturating the magnetization direction of the core in opposite directions is primarily used to reduce the noise contribution of the read out electronics by modulating the output signal of the fluxgate to frequencies above the 1/f knee frequency of these electronics. And, as discussed below, this is also an important aspect in reducing the magnetic noise from the core of the fluxgate.

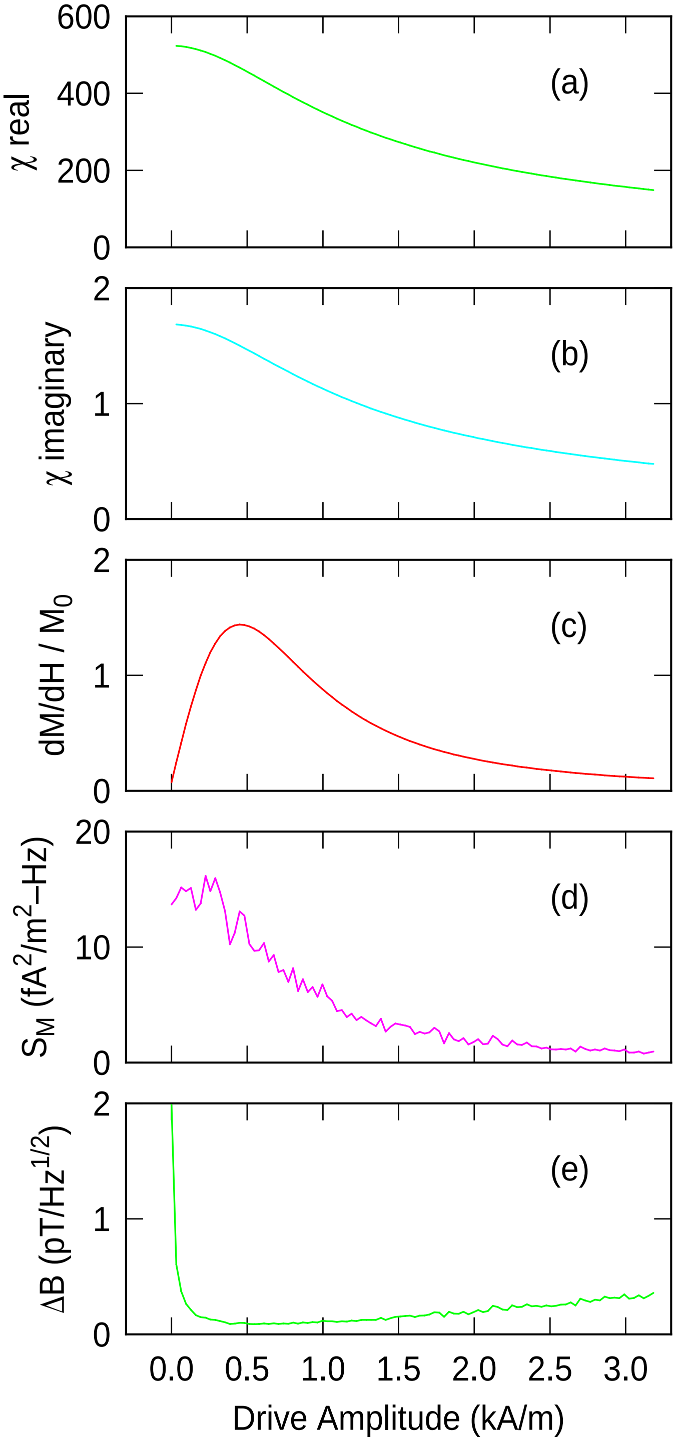

FIG. 2. (a)Real part of averaged susceptibility $\Delta M/\Delta H$ at 1 MHz vs. drive amplitude for simulated fluxgate. (b)Imaginary part of averaged susceptibility. (c)dc response. (d)Magnetization noise at 1 kHz and (e)Field noise at 1 kHz. The hard axis bias field was 760 A/m. Both noise spectra were measured at the output of the simulated mixer.

The simplest approach for predicting the noise of the fluxgate ignores the effects of the modulation or drive field. We can use the magnetic susceptibility of the core to predict the equilibrium magnetization noise, S_M^{eq}, using the fluctuation dissipation theorem,

$$S_M^{eq} = \frac{4k_BT\chi''(\omega)}{\Omega\omega}, \qquad (1)$$

where χ'' is the lossy part of the susceptibility measured at zero field, T is the temperature, ω is the measurement frequency, and Ω is the core volume.

The prediction for the noise in Eq. 1 does not apply directly to the operation of the fluxgate, which is not in thermal equilibrium since the core is alternately saturated in opposite directions. Hence $< \chi >$ is reduced, Eq. 1 generally overestimates the white magnetic noise. To develop a model of the noise that explicitly includes the modulation field, we have used the LLG equation to understand how intrinsic thermal fluctuations of the core evolve in a non-linear way into fluctuations in M [5]. This type of strategy has worked well in other non-linear physical systems [6]. The noisy LLG equation is

$$d\vec{M}/dt = -\gamma\mu_0\vec{M}\times\vec{H} - \frac{\alpha\gamma\mu_0}{M}\vec{M}\times\vec{M}\times\vec{H} + \gamma\mu_0\vec{M}\times\vec{F}(t), \qquad (2)$$

where M is the sample magnetization, α is the Landau-Lifshitz loss parameter, and γ is the gyromagnetic ratio divided by $(1+\alpha^2)$. The applied field H includes the shape $\boldsymbol{d}$ and materials $\boldsymbol{k}$ anisotropy fields, $H_i = H_{i,ext} + k_iM_i + d_iM_i$ *for* $(i = x, y, z)$. $F(t)$ is a Langevin noise term that is Gaussian and δ-correlated in time with a power spectrum [5]

$$S_F = \frac{4k_BT\alpha}{\gamma\mu_0^2 M\Omega}. \qquad (3)$$

We solve Eq. 2 by integrating the equation in time and randomly choosing appropriate values of $F(t)$ [7]. We can estimate the dc M-H loop, as shown in Fig. 1, by simulating without an applied ac drive field. In order to estimate the responsivity and sensitivity, we then apply a sinusoidal magnetic field drive at a frequency ν_D to alternately saturate the core in either direction. The value of $\vec{M}(t)$ is recorded and is digitally mixed with a $2\nu_D$ sine wave reference signal. The value of the product is the output of the fluxgate. The power spectrum of that output, S_M, is the magnetization noise of the fluxgate. To estimate the responsivity of the fluxgate, we first fix the uniform dc field applied to the core to zero and record the output. Then we apply a small dc field ΔH to the core and again simulate. The change in output ΔM allows an estimate of dM/dH to be made. Finally, we have that the field noise S_B is given by

$$S_B = \mu_0^2 S_M/(dM/dH)^2. \quad (4)$$

Our fluxgate simulation directly estimates ΔM. A real fluxgate would measure the magnetization using the voltage generated across the pickup coil.

The fluxgate core is modeled as permalloy with $\mu_0 M = 1T$, and an easy axis anisotropy of $20\,J/m$ in the x direction, a 760 A/m hard axis bias field in the +y direction, and a temperature of 300 K. These parameters and the volume, $2.23 \times 10^{-8}\,m^3$ were chosen to correspond to our previously reported single domain fluxgate [4]. The sinusoidally varying drive field is applied in the +z direction at the drive frequency, 1 MHz, and the value of α was 1.

Fig. 2 plots the results of our simulations for the real and imaginary susceptibility, the responsivity dM_z/dH_z, the magnetization noise S_M, and the effective field noise S_B at a measurement frequency of 1 kHz. The susceptibilities are defined as $\Delta M_z/\Delta H_z$ measured at the drive frequency.

The minimum value of the field noise is $90\,fT/\sqrt{Hz}$ at 500 A/m. The minimum noise occurs at the maximum value of the responsivity, which for this core occurs at a drive-field value that roughly half saturates the core at the drive frequency and bias field. The value of the simulated magnetization fluctuations S_M at that point is $10^{-14} A^2/m^2 Hz$, which is considerably less than the value predicted by knowing χ and using Eq. 1. This is a key point.

Often real fluxgates operate at much larger values of drive field in order to reduce the 1/f noise. Indeed, the best noise level measured in our single domain tube fluxgate was larger, about $1.4\,pT/\sqrt{Hz}$ at 1 Hz, and limited by 1/f noise.

As α increases from 1, the minimum value of the field noise amplitude increases approximately linearly as α. As α is decreased below 1, the dynamics changes from a simple controlled up-down motion with the drive field, to a complex limit cycle covering most of the available phase space. The noise remains approximately at the $100\ fT/\sqrt{Hz}$ level for $\alpha\ \leq\ 1$ Near the simulated bias field and anisotropy there is a broad minimum in the noise performance. Some bias field is needed for the best sensitivity, e.g. a bias field of 76 A/m results in a minimum noise level of about $300\ fT/\sqrt{Hz}$. Also the drive field required to achieve the minimum noise increases as the anisotropy increases.

Often for the large macroscopic core of a real fluxgate, eddy currents contribute to the dynamics and the noise. For example if we imagine a core that is a tube of length l, outside radius r, and wall thickness t, the effective resistance R of the core for circumferential currents is $R = \rho 2\pi r/lt$, where ρ is the resistivity of the core material. The drive field creates an electromotive force around the core, $V = jH_D\mu\mu_0\pi r^2\omega_D$, where H_D and ω_D are the drive field and frequency and μ is the effective permeability of the core. The resulting current I around the core creates a field $H_{Eddy} = I/2l$ or

$$H_{Eddy} = jH_D\mu\mu_0 r\omega_D t/4\rho. \quad (5)$$

The phase of the field appears as an effective increase in α from typical values of 0.01 for ferromagnets to much larger values, $\mu\mu_0\omega_D rt/4\rho$. Measured values for bulk cores can range from 1 up to 1000 and beyond. The exact value depends on the eddy current mode generated by the drive. For high frequencies, the skin depth of the core must be considered. The possibility of creating eddy currents also means thermal noise from that resistance will create white field noise. For the example above, the Johnson noise currents from the core, $S_I = 2k_T Tlt/\pi\rho r$, create an effective H field noise

$$S_{H,Eddy} = k_B Tt/2\pi\rho rl. \quad (6)$$

Often the resulting B field noise from the eddy current loss will be larger than the noise associated with the intrinsic α term in the LLG equation.

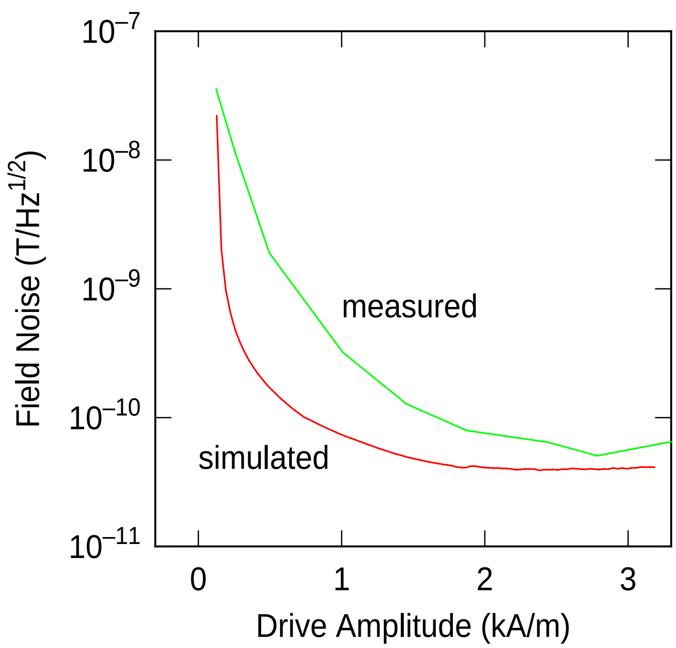

FIG. 3. Measured and simulated white fluxgate noise for thin film fluxgate. The white noise was measured at a frequency where the 1/f noise falls below the white noise level. The drive frequency for both the experiment and the simulation was 22 kHz. The permalloy film was 1 cm in diameter and 0.5 microns thick.

Fig. 3 compares the measured value of the white noise for our thin film fluxgate with the results predicted from our numerical model using the LLG equation. The fluxgate is modeled as a permalloy film with a volume of $1.8 \times 10^{-10}\,m^3$, an anisotropy of $1.3\,J/m$ and an α of 30, all of which are consistent with independent susceptibility measurements of these parameters. The large value of α for this film indicates that eddy current loss dominates the intrinsic loss mechanism for the permalloy. The actual path of the eddy currents for this geometry is complicated, but using scaling arguments similar to those in Eq. 5 above, the effective value of α is $\mu\mu_0\omega_D tr/\rho$ or $\sim 1-10$, which is consistent with the measurements. Despite a factor of three difference between the measured noise when the drive amplitude is near 1000 kA/m, overall there is good agreement between the data and the model. We do not completely understand the origin of the difference. It may be a measure of the degree to which the actual core departs from single domain behavior during operation.

In conclusion we have presented a simple model for the limiting white noise of a fluxgate and experimental evidence of its validity. These results suggest that for a physically realizable fluxgate, the white noise is limited by the intrinsic loss in the core and $100\, fT/\sqrt{Hz}$ is possible. To achieve this performance, the core geometry and materials must be chosen to limit eddy currents. To date the lowest values of white noise measured are typically $1pT/\sqrt{Hz}$. In this paper we have not addressed the problem of 1/f noise in the fluxgate, which is usually the dominate noise source at low frequencies. Understanding and modeling 1/f noise remains a difficult future challenge.

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