Minimum Energy Reliable Paths Using Unreliable Wireless Links

Abstract

We address the problem of energy-efficient reliable wireless communication in the presence of unreliable or lossy wireless link layers in multi-hop wireless networks. Banerjee and Misra had attempted to address this problem in their prior work [1] but had provided an optimal energy-efficient solution to this problem for the case where link layers implement perfect reliability. However, a more common scenario — a link layer that is not perfectly reliable, was left as an open problem. In this paper we first present two centralized algorithms, BAMER and GAMER, that optimally solves the minimum energy reliable communication problem in presence of unreliable links. Subsequently we present a distributed algorithm, DAMER, that approximates the performance of the centralized algorithm and leads to significant performance improvement over best known single-path or multi-path based existing techniques.

1 Introduction

Wireless communication networks have been deployed at an increasingly fast rate, and are expected to reshape the way we live in this physical world. For example, wireless ad hoc networks combined with satellite data networks [15] are able to provide global information delivery services to users in remote locations that can not have been reached by traditional wired networks. Meanwhile, advances in hardware technology are constantly proliferating various wireless communication terminals (e.g., smart phones or PDAs) to an exploding user population. In many scenarios, design of wireless protocols are guided by two requirements — energy efficiency and resilience to packet losses. Efficiently handling losses in wireless environments, therefore, assumes significant importance. Even under benign conditions, various factors, like fading, interference, multi-path effects, and collisions, lead to heavy loss rates on wireless links [10, 9, 34, 7, 31, 33]. Due to the end-to-end reliability requirement of many applications, it is necessary to study how such reliability can be guaranteed in an energy efficient way in wireless environments. In this paper we examine the problem of energy efficient routing of traffic in a multi-hop wireless network that appropriately handles packet losses in the wireless environment.

There are two well-known ways to achieve end-to-end reliability on multi-hop paths. The first approach employs *hop-by-hop* retransmissions — each link layer hop retransmits lost frames as and when necessary. The second approach assumes that link layers are unreliable and retransmissions are performed *end-to-end.* It is also possible to consider a mix of the above as a third approach, where link layers perform a few retransmissions if necessary, but perfect reliability is only guaranteed through end-to-end mechanisms.

Traditional power aware routing schemes [22, 5] do not take link loss rates into account when computing energy efficient paths. By ignoring the impact of such losses, they implicitly assume that every link is totally reliable. That paradigm is obviously too optimistic, and retransmissions consume power as well. In order to achieve better energy efficiency in realistic scenarios, the right metric should be the cumulative energy consumption due to all packet transmissions including retransmissions.

Prior work by Banerjee and Misra [1] solved the problem of computing energy efficient paths for the hop-by-hop retransmission model only and had left optimal approaches for the end-to-end case as an open problem. However, all practical mechanisms to achieve perfect end-to-end reliability guarantees rely either on the end-to-end model or on the mixed approach (combination of hop-by-hop and end-to-end retransmissions). For example, link layer technologies such as the 802.11 MAC protocol [16] typically make a bounded number of retransmission attempts for a lost or corrupted frame. Further losses can be recovered through end-to-end retransmissions. The following are a summary of examples which underline the importance of energy-efficient solutions under the end-to-end and the mixed retransmission models:

- Link layer technologies such as IEEE 802.11 [16] typically implement a limited number of retransmissions, which results in possible delivery failure over lossy links.
- There are link level technologies that do not provide hop-by-hop retransmission (e.g. TRAMA [18]).
- Given link layer reliability, packet loss may still happen at network layer due to various reasons (e.g. congestion in WSNs [26]).

- Nodes may move, sleep, or fail. In such cases, hop-by-hop reliability can not be assumed. Note that even if a sleeping node can receive packets after waking up, the transport protocol may have timed out.

As long as there is some link in the multi-hop path that can not guarantee reliable packet delivery, we will have to rely on TCP-like transport protocols to initiate end-to-end retransmissions back from the source.

In this paper, we first solve the problem of computing minimum energy paths for reliable communication in the pure *end-to-end retransmission model* where none of the links in a wireless path guarantees any reliability. We next proceed to study the more general and realistic *mixed retransmission model* where some links may provide partial reliable delivery while the others may not. For example, even if the link level technology supports hop-by-hop retransmission, some links may still be unreliable due to other reasons described above. The BAMER and GAMER algorithms are designed for computing minimum energy paths in these models respectively. The hop-by-hop model and the pure end-to-end model are just special cases of the mixed model. Therefore, our algorithms for the mixed model can be used to find minimum energy paths in any network configurations.

For implementation in many practical scenarios, we may need a simple and light weight distributed protocol. In this paper, we also propose a distributed routing protocol, DAMER, for energy efficient routing in the general mixed retransmission model. Clearly, DAMER can be used in any network configuration, too. We show that DAMER is able to find the minimum energy path in the hop-by-hop retransmission model. And simulation results demonstrate that DAMER also effectively improves energy efficiency over the best known existing techniques in the general mixed retransmission model.

While the main focus of this paper is on single-path routing, we also examine the problem of reliability through utilization of multiple redundant paths. Prior work has examined the use of such multi-path routes in improving throughput or reliability [32, 23] mostly at the cost of increased energy consumption. To illustrate this aspect, we performed simulation-based comparisons of our technique with one of these prior techniques, GRAB [32] and the energy consumption of GRAB to achieve reasonable reliability is orders of magnitude larger than that of our schemes. Interestingly we found that by carefully choosing multi-path routes for data delivery it is possible to reduce energy consumption than best possible single-path route. In particular, we formally analyze the problem of finding the minimum energy multi-path routing scheme and prove that it is actually NP-hard. To the best of our knowledge, this paper is the first to investigate the potential of multi-path routing on energy conservation. Through extensive simulations, we demonstrate that our algorithms can significantly improve energy efficiency over best known existing techniques. Moreover, we carefully examine the effects of a number of network parameters on the performance of our algorithms as well as existing techniques. This study further enhances our understanding of energy efficient reliable communication in the presence of lossy links.

The rest of the paper is organized as follows. Section 2 reviews previous related work. Our network model and problem formulation are presented in Section 3. In Section 4, we present two algorithms as well as a distributed routing protocol for finding minimum energy paths in the mixed retransmission model. In Section 5, we examine multi-path routing as a potential means of energy conservation in the presence of unreliable links, and formally analyze its complexity. An empirical study through extensive simulations of our schemes as well as the best known current schemes is presented in Section 6. Finally, we conclude the paper in Section 7.

2 Related work

Energy efficient routing has always been a central research topic in wireless networks, both in the paradigm of multicast/broadcast [28, 27, 2, 14, 30, 29, 3, 8, 12] and in the paradigm of unicast [20, 22, 21, 4, 5, 24, 23]. In both paradigms, our objective is to design a routing scheme such that the total transmission power is minimized. In this paper, we study the paradigm of unicast and refer interested readers to the literature for more knowledge on energy efficient multicast/broadcast routing.

By using Dijkstra's shortest path algorithm, PAMAS [21] finds a minimum cost path where the link cost is set to the transmission power. If every link in the paths is error free, then a single transmission over each link can successfully deliver a packet from the source to its destination with a minimum energy consumption. Scott and Bamboos [20] studied the case where link costs include power consumption on the receiver side, and proposed to find energy efficient paths using a modified form of the Bellman-Ford algorithm [6].

Some researchers have considered power aware routing in an alternative approach. The residual batter power is used as a routing metric, in order to achieve a more balanced distribution of power consumption among all the nodes so that the lifetime of the whole system may be increased. From our perspective, these schemes may result in less energy efficient routes. We refer the reader to the literature [22, 4, 5, 24] for detailed information.

Unfortunately, none of these previous papers considered the lossy property of wireless links. Banerjee and Misra [1] explored the effect of lossy links on energy efficient routing and solved the problem of find minimum energy paths in the hop-by-hop retransmission model. Let w and p denote the transmission power and the error rate of a hop-by-hop retransmission link, respectively. [1] proposed the link cost to be $\frac{w}{1-p}$, which is actually the expected energy consumption on delivering a packet over that link [1]. For the hop-by-hop retransmission model it is then straightforward to use a traditional shortest path algorithm, e.g. Dijkstra's algorithm, to compute minimum energy paths.

The same is, however, not true in the end-to-end retransmission model. Therefore the authors in [1] only proposed an approximate heuristic that defines the link cost to be $\frac{w}{(1-p)^l}$, where $l \geq 2$ is some constant, and used Dijkstra's algorithm to compute low-energy paths. For simplicity, in this paper we denote Banerjee and Misra's algorithm by BMA and denote BMA with $l = k$ by BMA-k. In the end-to-end retransmission model, packet loss at intermediate links will abort the whole delivery thus far and incur end-to-end retransmissions back from the source, which means more transmission power is wasted than in the hop-by-hop model. Intuitively, $l \geq 2$ makes lossy links appear to be even more expensive. BMA-l thus prefers less lossy links and reduces the risk of incurring end-to-end retransmissions. While such a choice is reasonable, clearly it is not optimal. Additionally the more general and realistic mixed retransmission model is not explored in [1].

Multi-path routing has been proposed as a means of improving reliability as well as throughput. GRAB [32] forwards packets along an interleaved mesh, and controls the width of the mesh hence the success ratio by assigning an appropriate *credit* to each packet. We here point out that the multi-path scheme of GRAB harnesses the high redundancy and large scale of WSNs, and is not appropriate for other network models. In contrast, this paper considers a more general network model. Moreover, GRAB provides only robust delivery instead of reliable delivery, which means packets are not guaranteed to be delivered in GRAB. Srinivas and Modiano [23] investigate the problem of minimum energy node/link disjoint paths routing in multi-hop wireless networks. Clearly, such schemes result in increased energy consumption, compared with the minimum energy single path. And they do not provide guaranteed delivery, either. Again, none of them explicitly considers link error rates.

Transport protocols (e.g. PSFQ [25]) have also been proposed to provide reliable communication over unreliable wireless links. Unlike routing protocols, transport protocols do not pay attention to route selection hence are beyond the scope of this paper.

3 Formulation

In our network model, each network node is assumed to be equipped with an omnidirectional antenna. A wireless network is modelled as a directed graph $G = (V, A)$, where V is the set of nodes and A is the set of directed links. Each node is assigned a unique ID $i \in [1..|V|]$ and has a maximum transmission power of $P_{max}(i)$. Each directed link (i, j) has a non-negative *weight* $W(i, j)$, which denotes the minimum transmission power required to maintain a reasonably good quality link from node i to node j. Wireless propagation suffers severe attenuation [13, 17, 19]. Let d_{ij} denote the distance between node i and node j. If i transmits with power $P_t(i)$, the power of the signal received by node j is given by

$$P_r(j) = \frac{P_t(i)}{c \cdot d_{ij}^{\alpha}},$$

where α and c are both constants, and usually $2 \leq \alpha \leq 4$ [19]. In order to correctly decode the received signal at the receiver side, it is required that

$$P_r(j) \geq \beta_0 \cdot N_0,$$

where β_0 is the required *signal-to-noise ratio (SNR)* and N_0 is the strength of ambient noise. Thus, the weight of link (i, j) is given by

$$W(i, j) = c \cdot \beta_0 \cdot N_0 \cdot d_{ij}^{\alpha}. \tag{1}$$

Each link (i, j) also has an *error rate* (or *loss rate*) $Er(i, j)$, which is the probability that a transmission over link (i, j) does not succeed. If $Er(i, j) = 0$, link (i, j) is considered *reliable*. G contains link (i, j) if and only if $W(i, j) \leq P_{max}(i)$ and $Er(i, j) < 1$. The expected number of transmissions (including retransmissions) of a successful delivery over link (i, j) is given by

$$N(i, j) = \frac{1}{1 - Er(i, j)}. \tag{2}$$

Each node is capable of adjusting its transmission power according to the outgoing link weights, in order to conserve as much power as possible. Typically, energy efficient routing schemes tend to choose paths composed of a large number of short distance links since long distance links are much more power consuming given that $\alpha \geq 2$. Link failure is presumed to be independent and unpredictable, so the metric is defined to be the expected total energy consumption of a successful delivery. By *minimum energy path* from node u to node v, we refer to a path that has the minimum expected energy consumption of a successful delivery from u to v. Let $C_{min}(u, v)$ denote the expected energy consumption of a successful delivery along a minimum energy path from u to v.

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[1] A similar metric, ETX, was proposed by DeCouto *et al* [7] for computing high throughput paths.

We refer to the general problem of finding the minimum energy routing scheme in the mixed retransmission model as the *Minimum Energy Reliable Communication Using End-to-end Retransmissions* problem and formally define it as follows.

MINIMUM ENERGY RELIABLE COMMUNICATION USING END-TO-END RETRANSMISSIONS

INSTANCE Directed graph $G = (V, A)$. Link weight function $W : A \rightarrow R_0^+$. Link error rate function $Er : A \rightarrow [0, 1)$. Function $U : A \rightarrow \{0, 1\}$ indicates whether a link provides hop-by-hop retransmission. Specified source s and sink t. Non-negative bound B.

QUESTION Is there a routing scheme such that the expected energy consumption of a successful delivery from s to t is no more than B?

4 Single-path min-energy routes

In this section, we present a number of algorithms to compute minimum energy paths for reliable communication over lossy links in multi-hop wireless networks. We start by studying the seemingly simpler end-to-end retransmission model, for which we present the *Basic Algorithm for Minimum Energy Routing (BAMER)*. Then in Section 4.2, we study the more general and realistic mixed retransmission model. The *General Algorithm for Minimum Energy Routing (GAMER)* is proposed for that case. In Section 4.3, we show that an appropriate preprocessing stage enables BAMER to solve the same problem in the mixed model as well. While BAMER and GAMER are both centralized algorithms, typically routing needs to be carried out in a distributed fashion. Towards that end, we propose the *Distributed Algorithm for Minimum Energy Routing (DAMER)* in Section 4.4.

4.1 Basic Algorithm for Minimum Energy Routing (BAMER)

We first present BAMER and show that it finds minimum energy paths from s to all other nodes in the end-to-end retransmission model (Table 1).

Lemma 1 *Let $P(s, v)$ denote a minimum energy path from s to v, in which node u is the predecessor of v. The prefix part of $P(s, v)$ between s and u, denoted by $P(s, u)$, has to be a minimum energy path from s to u.*

For any path $P(s, v)$, Let $Cost(P(s, v))$ denote the expected energy consumption of that path. The key observation is that

$$Cost(P(s, v)) = N(u, v)[Cost(P(s, u)) + W(u, v)].$$

To prove by contradiction, assume that $P(s, u)$ is not a minimum energy path from s to u, while another path $P'(s, u)$ is a minimum energy path from s to u. We can simply replace $P(s, u)$ in $P(s, v)$ with $P'(s, u)$. The resulted new path from s to v, denoted by $P'(s, v)$, will have an expected energy consumption of

$$\begin{aligned} Cost(P'(s, v)) &= N(u, v)[Cost(P'(s, u)) + W(u, v)] \\ &< N(u, v)[Cost(P(s, u)) + W(u, v)] \\ &= Cost(P(s, v)). \end{aligned}$$

This contradicts the fact that $P(s, v)$ is a minimum energy path from s to v. Proof is completed. □

```
BAMER ($G$, $s$, $W$, $Er$, $N$, $T$, $C$)
for each node $v \in V(G)$ do
    $T(v) \leftarrow \phi$
    $C(v) \leftarrow \infty$
$C(s) \leftarrow 0$
$S \leftarrow \{s\}$
$u \leftarrow s$
while $S \neq V(G)$ do
    for each node $v \in V(G) - S$ do
        if $N(u, v)[C(u) + W(u, v)] < C(v)$
            $T(v) \leftarrow T(u) \cup \{(u, v)\}$
            $C(v) \leftarrow N(u, v)[C(u) + W(u, v)]$
    $u \leftarrow v \in V(G) - S$ s.t. C(v) is minimum
    $S \leftarrow S \cup \{u\}$
```

Table 1: Pseudo code description of BAMER. G is the directed graph. s is source. W and N are defined in Equations 1 and 2 respectively. T contains the edges on the minimum energy path and $C(v)$ is the cost of the computed path from the source to the current node v.

Lemma 2 *In BAMER, each time a node v is added to S, links in $T(v)$ form a minimum energy path from s to v hence $C(v) = C_{min}(s, v)$.*

We prove Lemma 2 by induction on the order of nodes being added to S. The base case is trivially true. Now assume that Lemma 2 holds for every node already in S, and a node v is then chosen to be added to S. Consider any minimum energy path $P(s, v)$ from s to v.

If all previous nodes in $P(s, v)$ have been in S, by Lemma 1 and inductive assumption it is clear from the description of BAMER that

$$C(v) \leq Cost(P(s, v)) = C_{min}(s, v).$$

If at least one previous node in $P(s, v)$ has not been in S yet, let u denote the first such previous node in $P(s, v)$ (counting from s to v). And let $P(s, u)$ denote the prefix part of $P(s, v)$ between s and u. By Lemma 1, it is clear from the description of BAMER that

$$C(u) \leq Cost(P(s, u)) = C_{min}(s, u).$$

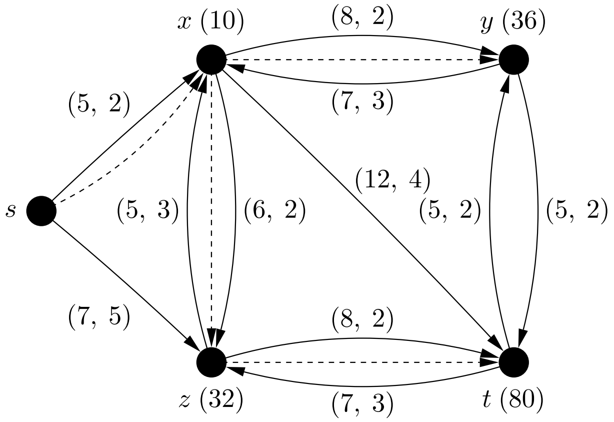


Figure 1: Illustration of BAMER

Given that BAMER chose v instead of u, it is the case that

$$C(v) \leq C(u) \leq C_{min}(s,u) \leq C_{min}(s,v)$$

since u is a previous node in $P(s,v)$.
Let u' be the node already in S that assigns $C(v)$ to v in BAMER. By inductive assumption, links in $T(u')$ form a path from s to u whose expected energy consumption is $C(u)$. Thus, $T(v) = T(u') \cup \{(u',v)\}$ form a path from s to v, and the expected energy consumption is $C(v)$. Since we have proved that $C(v) \leq C_{min}(s,v)$, it has to be the case that $C(v) = C_{min}(s,v)$ and links in $T(v)$ form a minimum energy path from s to v. Proof is completed. □

Corollary 1 *For each node $v \in V(G)$, BAMER computes a minimum energy path from s to v.*

We illustrate BAMER with the example in Figure 1. In the example network, each link (u,v) is labelled with the $(W(u,v), N(u,v))$ pair, and each node u is labelled with its ID and $C(u)$. x is the first node added to S by BAMER, followed by its successors z and y in order. BAMER terminates after choosing t, whose predecessor is z. The minimum energy paths are indicated by the dashed links. The minimum expected energy consumption to deliver a packet from s to t is 80. BMA-1 will choose the path $s \rightarrow x \rightarrow y \rightarrow t$ and the expected energy consumption is 82. Without considering link loss rates, a naive shortest path algorithm (e.g. Dijkstra's algorithm) will choose the path $s \rightarrow z \rightarrow t$, incurring an expected energy consumption of 86.

4.2 General Algorithm for Minimum Energy Routing (GAMER)

In Section 4.1, we present the BAMER algorithm for finding minimum energy paths in the pure end-to-end retransmission model where no link guarantees per hop reliability through hop-by-hop retransmissions. This is in contrast to prior work (BMA) which solved the problem in the idealized model where each link is perfectly reliable. In realistic scenarios, we may have to solve the minimum energy path problem in the more general mixed retransmission model, where different point-to-point links are implemented with different link level technologies, or other factors may make some links unreliable in the presence of inherently reliable link level technologies, etc. In this section, we solve the minimum energy path problem in this mixed retransmission model with our *General Algorithm for Minimum Energy Path (GAMER)*, which is described in Table 2.

```
GAMER ($G$, $s$, $W$, $Er$, $N$, $T$, $C$)
for each node $v \in V(G)$ do
    $T(v) \leftarrow \phi$
    $C(v) \leftarrow \infty$
$C(s) \leftarrow 0$
$S \leftarrow \{s\}$
$u \leftarrow s$
while $S \neq V(G)$ do
    for each node $v \in V(G) - S$ do
        if $(u,v)$ provides link layer retransmission
            if $C(u) + N(u,v)W(u,v) < C(v)$
                $T(v) \leftarrow T(u) \cup \{(u,v)\}$
                $C(v) \leftarrow C(u) + N(u,v)W(u,v)$
        else if $N(u,v)[C(u) + W(u,v)] < C(v)$
                $T(v) \leftarrow T(u) \cup \{(u,v)\}$
                $C(v) \leftarrow N(u,v)[C(u) + W(u,v)]$
    $u \leftarrow v \in V(G) - S$ s.t. C(v) is minimum
    $S \leftarrow S \cup \{u\}$
```

Table 2: Pseudo code description of GAMER. Parameters same as in Table 1

We show that Lemma 1 also holds for GAMER. The case where (u,v) does not support hop-by-hop retransmission has been proved in Section 4.1. Now consider the case where (u,v) supports hop-by-hop retransmission. The observation is that

$$Cost(P(s,v)) = Cost(P(s,u)) + N(u,v)W(u,v).$$

To prove by contradiction, assume that $P(s,u)$ is not a minimum energy path from s to u, while another path $P'(s,u)$ is a minimum energy path from s to u. We can replace $P(s,u)$ in $P(s,v)$ with $P'(s,u)$. The resulted path $P'(s,v)$ will have an expected energy consumption of

$$\begin{aligned} Cost(P'(s,v)) &= Cost(P'(s,u)) + N(u,v)W(u,v) \\ &< Cost(P(s,u)) + N(u,v)W(u,v) \\ &= Cost(P(s,v)). \end{aligned}$$

This contradicts the fact that $P(s,v)$ is a minimum energy path from s to v. Proof is completed. □
Lemma 2 and its proof in Section 4.1 also hold for GAMER. This is easy to verify and we leave the details to the reader.

Corollary 2 *For each node $v \in V(G)$, GAMER computes a minimum energy path from s to v.*

To illustrate how GAMER works, let us return to to example in Figure 1. Now the link from x to t has been upgraded to support hop-by-hop retransmission. This does not change the behavior of traditional shortest path algorithms and BMA. However, GAMER will find the minimum energy path $s \rightarrow x \rightarrow t$ and the expected energy consumption goes down from 80 to 58.

4.3 BAMER for the mixed retransmission model

Although BAMER is motivated by and designed for the pure end-to-end retransmission model, it turns out an appropriate preprocessing stage will enable BAMER to solve the same problem in the mixed retransmission model. To see why and how, note that GAMER differs from BAMER only in lines 9–12 of Table 2, i.e., the case where link (u, v) supports hop-by-hop retransmission. Particularly, the only difference that matters is line 12. Note that the right side of line 12 can be viewed as $[C(u) + N(u, v)W(u, v)] \times 1$. Compared to the right side of line 11 in Table 1, we can see that link (u, v) can be treated as a reliable link that does not support hop-by-hop retransmission and has a new weight of $N(u, v)W(u, v)$. Therefore, we can preprocess the links that support hop-by-hop retransmission as is described above. Then, applying BAMER on the preprocessed network graph is provably correct to compute a minimum energy path from s to each node in the network.

To illustrate how BAMER works in the mixed retransmission model, we return to the example in Section 4.2. In the preprocessing stage, the point-to-point link (x, t) is marked with $(48, 1)$ as a link that does not support hop-by-hop retransmission and has a weight of 48. BAMER is then executed on the preprocessed network graph and correctly finds the minimum energy path $s \rightarrow x \rightarrow t$.

4.4 Distributed Algorithm for Minimum Energy Routing (DAMER)

Both BAMER and GAMER are centralized algorithms. In many applications, a routing algorithm has to be implemented as a distributed routing protocol. We here propose the *Distributed Algorithm for Minimum Energy Routing (DAMER)* for such applications. A pseudo code description of DAMER is presented in Table 3.

Observation 1 *For any node $w \in V(G)$, $C(w)$ never grows during the execution of DAMER.*

Observation 2 *In the hop-by-hop retransmission model, DAMER finds a minimum energy path to every node $t \in V(G)$ as BMA-1 does.*

```
     DAMER ($G$, $W$, $N$, $Nexthop$, $R$, $C$)
     /* initialization */
 1   for each node $v \in V(G)$ do
 2       $R(v) \leftarrow \infty$
 3       $C(v) \leftarrow \infty$
 4   $R(u) \leftarrow 1$
 5   $C(u) \leftarrow 0$
     /* periodic route exchange */
 6   for each round of route exchange do
 7       broadcast $R$ and $C$ in a route exchange message $M_u$
 8       for each neighbor $v$ do
 9           collect a route exchange message $M_v$ from $v$
10           for each node $w \in V(G)$ do
11               if $M_v.C(w) + M_v.R(w)N(u,v)W(u,v) < C(w)$
12                   $Nexthop(w) \leftarrow v$
13                   $C(w) \leftarrow M_v.C(w) + M_v.R(w)N(u,v)W(u,v)$
14                   if $(u,v)$ provides hop-by-hop retransmission
15                       $R(w) \leftarrow M_v.R(w)$
16                   else $R(w) \leftarrow N(u,v)M_v.R(v)$
```

Table 3: DAMER running at individual nodes. u represents the local node itself. For any node $w \in V(G)$, $R(w)$ records the expected number of end-to-end transmissions (including retransmissions) required to deliver a packet from u to w via $Nexthop(w)$. The expected energy consumption is recorded in C.

The verifications are easy and are omitted due to space constraints.

Lemma 3 *For any node $w \in V(G)$, whenever v is a downstream node on the path from u to w, it has to be the case that $C(w) > C_v(w)$ where $C_v(w)$ denotes $C(w)$ at node v then.*

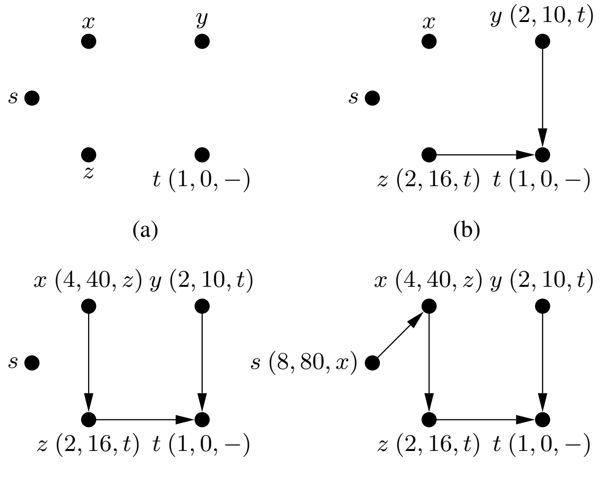
By Observation 1, it is clear from lines $11 \sim 13$ of Table 3 that whenever $Nexthop(w) = v$, it must be the case that $C(w) > C_v(w)$. Recursively applying this rule finishes the proof of Lemma 3. Proof is completed. □

Corollary 3 *Routes generated by DAMER are loop free.*

Based on the example in Section 4.2, we illustrate in Figure 2 the round by round execution of DAMER in finding an energy efficient path from s to t. Although we only illustrate a single *source-sink* pair here, we point out that DAMER actually finds such a path for every *source-sink* pair in the network.

5 Multi-path min-energy routes

In Section 4, we have proposed and proved BAMER and GAMER for computing the minimum energy path for reliable communication in multi-hop wireless networks. In-



(a) (b) (c) (d)

Figure 2: Illustration of DAMER. Each node is labelled with a $(R(t), C(t), Nexthop(t))$ tuple.

terestingly, we here point out that in some cases a multi-path routing scheme actually minimizes the expected energy consumption. Traditionally, multi-path routing is considered beneficial for improved throughput and reliability [32, 23]. Intuitively, improved throughput and reliability come at the cost a higher energy consumption due to the use of multiple (not necessarily disjoint) paths simultaneously. Therefore, it is not surprising that researchers have been designing single path routing algorithms for energy efficient one-to-one communication, as we do in Section 4. In this section, we reveal the interesting and counter-intuition fact that multi-path routing can potentially reduce the expected energy consumption of one-to-one reliable communication in the presence of unreliable links. Moreover, we formally analyze the complexity of finding the minimum energy routing scheme. To the best of our knowledge, this paper is the first to study exploiting multi-path routing to reduce energy consumption.

With an omnidirectional antenna, a single wireless transmission by a node can be received by every node within its transmission range. This property of wireless media is referred to as *Wireless Multicast Advantage (WMA)* [28]. WMA has been extensively studied in energy efficient one-to-many communication, e.g. minimum energy broadcast in wireless networks [28, 27, 2, 14, 3, 8]. We show that WMA and the use of multiple paths enable us to reduce energy consumption in one-to-one communication over unreliable links as well. Consider the example in Figure 3, where s needs to communicate with t. Links coming out of s have a loss rate of $\frac{1}{2}$ and a weight of 1. Links coming out of b_1 and b_2 are reliable and free. Consider the multiple paths routing scheme where every link participates. The probability of a successful delivery from s to t is $\frac{7}{8}$, and the expected energy consumption of a successful delivery is thus $\frac{8}{7}$. On the other hand, the expected energy consumption of any minimum energy single path is 2.

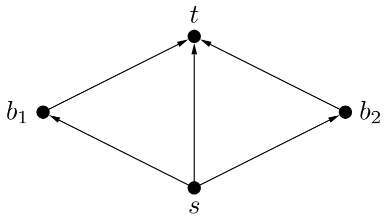


Figure 3: Wireless Multicast Advantage (WMA)

In a multi-path routing scheme, an intermediate node may receive multiple copies of the same packet from upstream nodes. Before we can proceed to formally analyze multi-path routing schemes, a problem that has to be answered is *"Should the intermediate node forward every copy of the packet?"* We believe the correct answer should be "No". Because forwarding the same packet more than once will incur unnecessary additional energy consumption at the intermediate node as well as downstream nodes, without knowing if that really helps at all.

We formally analyze the complexity of finding minimum energy multi-path routes in the Appendix and show that it is NP-Hard.

6 Simulations

We conduct extensive simulations in our empirical study in order to answer the following questions. Compared with the best known current schemes, how effectively can our algorithms conserve energy in a variety of network environments? How network parameters affect the performance of existing algorithms and ours? Such parameters include link error rates, value of α, percentage of links supporting hop-by-hop retransmission, network size (i.e., node population), and so on. Before we proceed to present the simulation results, we start by describing some technical details of our simulations.

In our simulations, 100 nodes of the same transmission range are distributed into a 10×10 square field uniformly at random. Two nodes are connected if and only if the distance between them is no larger than their transmission range. For each directed link, its link error rate is chosen from $[0, MaxLER]$ uniformly at random, where $0 \leq MaxLER \leq 1$ represents the *maximum link error rate*. Consequently, link (u, v) and link (v, u) may have different error rates. For each parameter setting, 1000 such trial networks are generated. In each trial network, we randomly pick a source node and a destination node. The average energy consumption of the paths computed in all 1000 networks is calculated for individual algorithms,

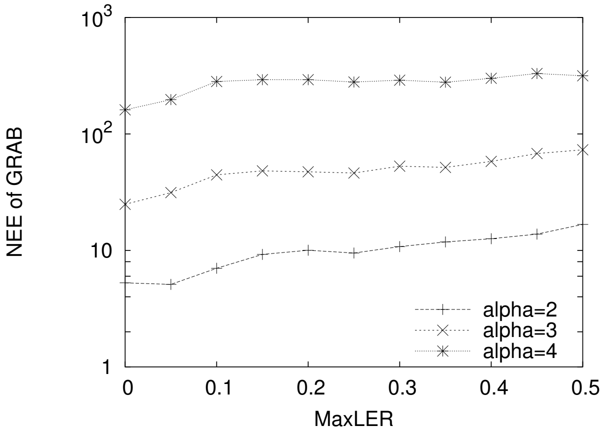


Figure 4: **Energy efficiency of GRAB normalized with respect to BAMER and GAMER.**

respectively. To evaluate the effectiveness of our algorithms in conserving energy, we define *normalized energy efficiency (NEE)* of an algorithm to be the ratio of its average energy consumption to that of BAMER and GAMER, since BAMER and GAMER are guaranteed to find a minimum energy path.

For single path routing, we compare our algorithms with the best known BMA algorithm. For multi-path routing, we compare with GRAB [32], as node/link disjoint paths [23] clearly consume more energy than the minimum energy single path. GRAB claims to be more efficient and flexible than disjoint paths in that it forwards packets along an interleaved mesh, and controls the width of the mesh by assigning an appropriate credit to each packet. We first conduct simulations in the end-to-end retransmission model to compare the energy efficiency of our algorithms and GRAB, since GRAB assumes the unreliable CSMA MAC. Figure 4 demonstrates that the energy consumption of GRAB is typically some orders of magnitudes larger, in order to achieve a delivery ratio of 95%. For higher link error rates, this delivery ratio of 95% is not even achievable. Given this huge performance gap, we only compare with the best known single path routing scheme, BMA, in the sequel.

6.1 Effects of α and link error rates

We first examine the effects of link error rates and α on the energy efficiency of the algorithms we study. To fully understand the behavior of these algorithms in the general end-to-end retransmission model, we here investigate the case where none of the links supports hop-by-hop retransmission. We conduct extensive simulations for a number of different values of $MaxLER$, α, and l, and present the simulation results in Figure 5.

It is clear from Figure 5 that high link error rates generally emphasize the effectiveness of our algorithms. Because a higher link error rate means a higher probability of aborting the end-to-end delivery done thus far and restarting a new end-to-end delivery back from the source. Thus, the performance of the relatively less intelligent BMA algorithms are more subject to link error rates.

Large α values demonstrate the same effect. Because large α values make short distance links even cheaper. Consequently, the algorithms tend to choose paths composed of more and shorter links. The more links a packet has to go through, the more likely that its delivery may fail and abort at some intermediate link. This means more energy consumption due to delivery abortions and end-to-end retransmissions.

Another clear message from Figure 5 is that reasonably large values of l consistently help BMA achieve better performance. Because large l values make lossy links appear to be prohibitively expensive to BMA. Consequently, BMA prefers less lossy links and that reduces the risk of delivery abortion. We also conduct simulations for $l > 4$, but typically that does not help conserve more energy. For eligibility, we only present simulation results for $1 \leq l \leq 4$. We will see the reason underlying this decision in later sections.

Finally, we point out that DAMER performs consistently better than BMA in the end-to-end retransmission model.

6.2 Effect of hop-by-hop retransmission

We have discussed in Section 6.1 that large l values help BMA conserve energy by avoiding lossy links. Clearly there has to be a cost to this trick. For example, consider the hop-by-hop retransmission model. $l = 1$ finds minimum energy paths, while larger values of l may give us less energy efficient paths. Intuitively, there should be some correlation between the optimal value of l and the percentage of links supporting hop-by-hop retransmission, which is denoted by $UPGrate$. We here reveal this correlation by conducting extensive simulations for a number of different values of l, $UPGrate$, and $MaxLER$. We assume a moderate setting of $\alpha = 2$, which is in favor of BMA algorithms as is shown in Figure 5. Simulation results presented in Figure 6 lead us to the following conclusions.

First, large l values perform better in the presence of a low $UPGrate$, while small l values perform better if a significant portion of links support hop-by-hop retransmission. Second, simulation results demonstrate that $l > 4$ does not help BMA. Depending on $UPGrate$ and $MaxLER$, $l = 3$ or 4 turn out to be the best choice. Third, by comparing different $MaxLER$ values, we can see that high link error rates are in favor of large values of l. This further verifies our previous understanding of the reason why large l values help BMA in the end-to-end retransmission

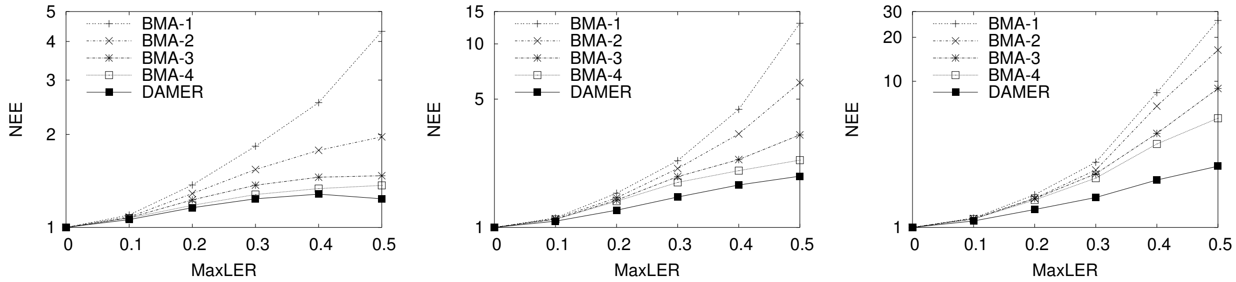


Figure 5: **Effects of α and link error rates on normalized energy efficiency (NEE). Normalization is with respect to GAMER, which finds the optimal paths. The figures represent $\alpha = 2, 3, 4$, respectively.**

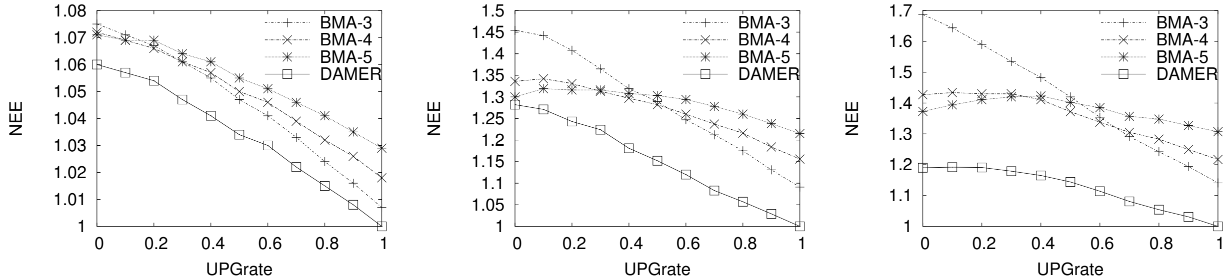


Figure 6: **Effect of hop-by-hop retransmission on normalized energy efficiency (NEE). Normalization is with respect to GAMER, which finds optimal paths. The figures represent $MaxLER = 0.1, 0.4, 0.7$, respectively.**

model: "pessimistic" estimations (i.e., large l values) better help BMA avoid high risk links (i.e., high error rates). Finally, even with the optimal setting of $l = 4$ and a moderate $\alpha = 2$, BMA still consumes more energy than BAMER and GAMER by up to 43%, and consumes more energy than DAMER by up to 22%.

Hop-by-hop retransmission consistently helps DAMER. In fact, we have discussed that DAMER is able to find minimum energy paths in the hop-by-hop retransmission model, and this is verified by the simulation results in Figure 6.

6.3 Effect of network size

As we have discussed in Section 6.1, the more links a packet has to go through, the more likely that its delivery may abort at some intermediate link. Since a larger network size (i.e., node population) leads to longer paths, the risk of delivery abortion will go up with network size. Accordingly, BMA needs to be more "pessimistic" on estimating link error rates so that it will further avoid lossy links to improve energy efficiency in the presence of increased network size. We here present an empirical investigation of the correlation between network size and l, as well as the effect of network size on the energy efficiency of DAMER and BMA. For consistency, we still assume that $\alpha = 2$. We conduct extensive simulations for a number of different values of network size, l, and $UPGrate$. Simulation results are presented in Figure 7.

As is shown in Figure 7, increased network size requires larger values of l. Meanwhile, increased network size also results in a lower energy efficiency of BMA. For example, when we have 30 nodes in the network, $l = 3$ is the best performing setting and it consumes up to 34% more energy than BAMER and GAMER, and consumes up to 28% more energy than DAMER. When we have 250 nodes, $l = 5$ is generally the best choice, which consumes up to 60% more energy than BAMER and GAMER, and consumes up to 35% more energy than DAMER. This fact draws our attention to an even more challenging problem of BMA: *without a priori knowledge of network size, how should BMA predetermine its optimal setting of l?* As is demonstrated by the simulation results, inappropriate l values can result in significantly lower energy efficiency of BMA, while our algorithms do not have this problem. For example, if BMA expects the network size to be 30 while the actual size is 250, it will consume up to 2.7 times the energy consumption of BAMER and GAMER, and consume up to 2.1 times that of DAMER.

7 Conclusions

In this paper, we study the problem of minimum energy routing for reliable communication in the presence of lossy links. Banerjee and Misra [1] solved the problem

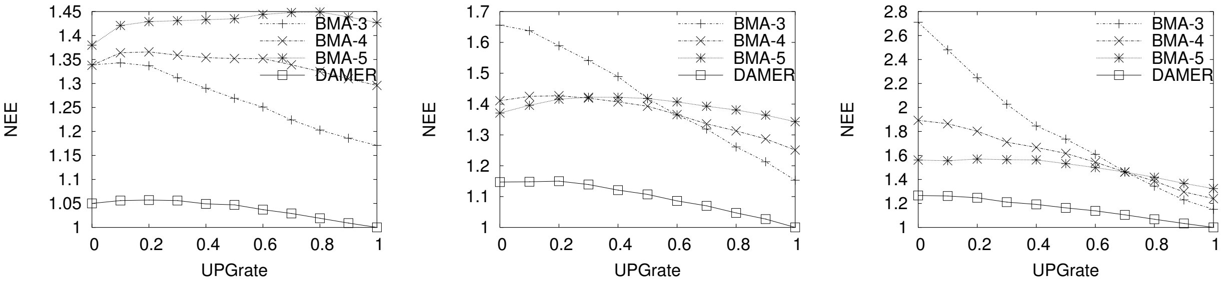


Figure 7: **Normalized energy efficiency variations with changes in network size. The figures represent 30, 150, 250 nodes, respectively.**

in the hop-by-hop retransmission model, where each link is assumed to support link layer hop-by-hop retransmission and guarantee reliable delivery. However, link layer retransmission actually can not guarantee reliable delivery, due to various reasons. In the end-to-end retransmission model where some link in the communication path is unreliable, we rely on TCP-like transport protocols to initiate end-to-end retransmissions. We first study the pure end-to-end retransmission model where none of the links guarantees per hop reliability, and then proceed to study the more general mixed retransmission model where some links may guarantee reliable delivery while the others may not. The BAMER and GAMER algorithms are designed for computing minimum energy paths in both models. The hop-by-hop model and the pure end-to-end model are just special cases of the mixed model, so BAMER and GAMER can be used to find minimum energy paths in any network configuration. For implementation in many practical scenarios, we also propose a light weight distributed routing protocol, DAMER, which can be used for energy efficient routing in any network configuration as well. DAMER is able to find minimum energy paths in the hop-by-hop model, and simulation results demonstrate that DAMER also effectively improves energy efficiency over the best known existing techniques in the general mixed model. Through extensive simulations, we also carefully examine the effects of a number of network parameters on the performance of our algorithms as well as existing techniques. This study further enhances our understanding of energy efficient reliable communication in the presence of lossy links.

Traditionally, multi-path routing have been utilized to improve throughput or reliability, possibly at the cost of increased energy consumption. Our another interesting finding is that, in some cases multi-path routing may reduce the expected energy consumption in the presence of lossy links. We formally analyze the problem of finding the minimum energy routing scheme and prove that it is actually NP-hard. To the best of our knowledge, this paper is the first to investigate the potential of multi-path routing on energy conservation.

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Appendix: NP-Hardness of Min-energy multi-path Routes

We formally analyze the complexity of finding the minimum energy multi-hop routes. We prove that it is NP-hard by reducing from the *3-dimensional matching (3DM)* problem, which is known to be NP-hard [11] and formally defined as follows.

3-DIMENSIONAL MATCHING (3DM)

INSTANCE Set $M = \{m_1, m_2, \ldots, m_n\} \subseteq W \times X \times Y$, where $W = \{w_1, w_2, \ldots, w_q\}$, $X = \{x_1, x_2, \ldots, x_q\}$, and $Y = \{y_1, y_2, \ldots, y_q\}$ are disjoint sets having the same number q of elements.

QUESTION Does M contain a matching, i.e., a subset $M' = \{m'_1, m'_2, \ldots, m'_q\} \subseteq M$ such that $|M'| = q$ and no two elements of M' agree in any coordinate?

Given an instance of 3DM, we construct a graph as shown in Figure 8, where nodes are distributed into four layers and edges exist only between nodes in adjacent layers. The graph in Figure 8 is constructed from the following

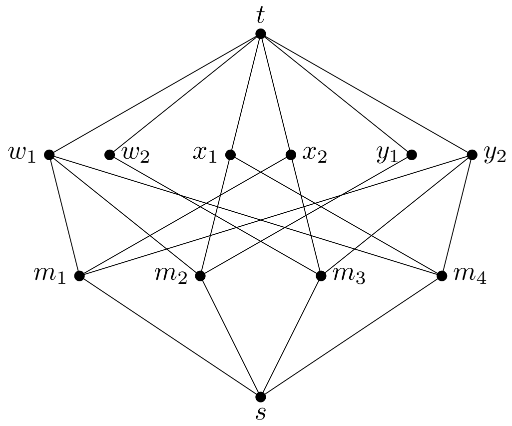

Figure 8: Reduction from 3DM

instance of 3DM.

$$\begin{array}{l} W=\{w_1,w_2\}, X=\{x_1,x_2\}, Y=\{y_1,y_2\}. \\ M=\{m_1,m_2,m_3,m_4\}. \\ m_1=(w_1,x_2,y_2), m_2=(w_1,x_1,y_1), \\ m_3=(w_2,x_2,y_2), m_4=(w_1,x_1,y_2). \end{array}$$

The top layer contains only the sink node t. In the second layer, there are three disjoint groups of *element nodes*, $\mathbb{W}=\{w_1,w_2,\ldots,w_q\}$, $\mathbb{X}=\{x_1,x_2,\ldots,x_q\}$, and $\mathbb{Y}=\{y_1,y_2,\ldots,y_q\}$, representing W, X, and Y, respectively. Each element node is connected to t with an edge whose weight is 0 and error rate is $p=e^{-1/3q}$. In the third layer, there are a set $\mathbb{M}=\{m_1,m_2,\ldots,m_n\}$ of *triplet nodes* representing the n elements of M. Each triplet node is adjacent to the three associated element nodes. Edges between element nodes and triplet nodes have a weight of 1 and an error rate of 0. The bottom layer contains only the source node s, which is adjacent to all triplet nodes. Edges between triplet nodes and s have a weight of $c=(e-1)q$ and an error rate of 0.

The transformation is polynomial, and we here show that M contains a 3-dimensional matching of size q if and only if the minimum expected energy consumption to deliver a packet from s to t is

$$\frac{c+q}{1-p^{3q}}=\frac{e^2q}{e-1}. \tag{3}$$

We start with the "only if" direction. If M contains a matching of size q, we can route a packet from s to t as follows.

- s transmits the packet to all the q triplet nodes contained in the matching.
- Each triplet node in the matching forwards the packet to its adjacent element nodes.
- Each element node forwards the packet to t.

The energy required to route the packet from s to the $3q$ element nodes is deterministically $c+q$. The probability that at least one element node successfully deliver the packet to t is $1-p^{3q}$. Thus, the expected energy consumption is given by (3).

We then prove the "if" direction. In particular, we show that the scheme described in the proof of the "only if" direction is the only scheme that can successfully deliver the packet at an expected energy consumption of (3). First of all, we point out that any routing scheme can be denoted by its number of forwarding triplet nodes $1\le n_0\le n$ and its number of forwarding element nodes $1\le q_0\le min(3q,3n_0)$. We prove by contradiction, assuming that M does not contain a matching of size q.

- If $n_0>q$, the expected energy consumption is
$$\frac{c+n_0}{1-p^{q_0}}>\frac{c+q}{1-p^{q_0}}>\frac{c+q}{1-p^{3q}}.$$
- If $n_0=q$, it has to be the case that $q_0<3q$ since we assume that M does not contain matching of size q. Thus, the expected energy consumption is
$$\frac{c+q}{1-p^{q_0}}>\frac{c+q}{1-p^{3q}}.$$
- If $n_0<q$ then $q_0\le min(3q,3n_0)=3n_0$. The expected energy consumption is thus
$$\frac{c+n_0}{1-p^{q_0}}\ge\frac{c+n_0}{1-p^{3n_0}}.$$

To conclude our proof by contradiction, it only remains to prove that,

$$\text{for any } 1\le n_0<q,\quad \frac{c+n_0}{1-p^{3n_0}}>\frac{c+q}{1-p^{3q}}. \tag{4}$$

On one hand, for any $x\ge1$,

$$\left[p^{3x}-(c+x)p^{3x}\ln p^3\right]'=-(\ln p^3)^2(c+x)p^{3x}<0.$$

On the other hand,

$$\begin{aligned} & p^{3x}-(c+x)p^{3x}\ln p^3|_{x=q} \\ = \;& (e^{-\frac{1}{3q}})^{3q}-[(e-1)q+q](e^{-\frac{1}{3q}})^{3q}\ln(e^{-\frac{1}{3q}})^3 \\ = \;& \frac{1}{e}+1 \\ > \;& 1. \end{aligned}$$

Therefore, for any x such that $1\le x\le q$,

$$\begin{aligned} & p^{3x}-(c+x)p^{3x}\ln p^3>1 \\ \iff\; & \frac{1-p^{3x}+(c+x)p^{3x}\ln p^3}{(1-p^{3x})^2}<0 \\ \iff\; & \left(\frac{c+x}{1-p^{3x}}\right)'<0. \end{aligned}$$

This completes the proof of (4). □