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Turbo Coded Continuous Phase Modulation

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Abstract

This paper investigates the performance of turbo coded continuous phase modulation (CPM). The proposed system uses standard binary turbo codes on M-ary input CPM channels by applying bit-interleaved coded-modulation (BICM), and thereby converting CPM channels into binary-input channels. Simulation results presented in this paper show that the turbo coded CPM signaling outperforms various other coding techniques for CPM signaling which are based on convolutional codes. The performance of the CPM signaling is compared to that of the phase-shift-keying (PSK) signaling and it is shown that the CPM signaling provides a more favorable trade-off between bandwidth and energy.

1. Introduction

The objective of this paper is to investigate the performance of turbo coded CPM signaling and compare it to the performance of turbo coded PSK signaling. Note that both CPM and PSK signals have constant envelope and are attractive for radio links with nonlinear transmitter amplifiers. While the performance of turbo coded PSK signaling is well documented, the same is not true for turbo coded CPM signaling. This paper indicates that a simple transmitter and receiver design can exploit the full potential of turbo codes on CPM channels.

2. System Architecture

The proposed transmitter and receiver use bit interleaved coded modulation (BICM) to convert an M-ary input CPM channel into several parallel binary-input channels [1]. Figure 1 shows the block diagram of the proposed communication system. In this setup, the binary codeword at the output of the channel encoder is fed to a bit interleaver π. The output of bit interleaver π is passed to a (one-to-one) labeling mapper μ which maps a group of Nk bits to a sequence of N symbol in an $M = 2^k$-ary input alphabet $\mathcal{X}$. Denote the resulting sequence of N symbols by $\mathbf{x}$. The sequence $\mathbf{x}$ is transmitted using continuous phase modulation.

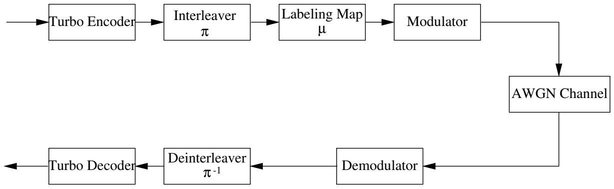

Figure 1: A turbo coded and continuous phase modulated communication system

The received signal $\mathbf{y}$ is passed to a demodulator which computes the bitwise soft information $p(\mathbf{y}|l^i(\mathbf{x}) = b)$. Here $l^i(\mathbf{x})$ denotes the i-th bit of $\mu^{-1}(\mathbf{x})$ and b belongs to the set $\{0, 1\}$. The demodulator which computes the bitwise soft information, is a straight-forward result of combining Rimoldi's time-invariant trellis of CPM signals [2] with the BCJR algorithm [3].

Finally, the bitwise soft information is deinterleaved and is passed to an iterative decoder. The analysis of BICM by Caire, Taricco, and Biglieri [1] shows that under the assumption of ideal interleaving, above arrangement converts an M-ary input CPM channel into Nk parallel, independent, memoryless, binary-input channels. These channels have roughly the same SNR and noise characteristics.

3. Simulation Issues and Results

In order to demonstrate the performance of turbo coded CPM channels, we used classical binary turbo codes with identical constituent convolutional codes generated by the polynomials $\{D^4 + D^3 + D^2 + D^1 + 1, D^4 + 1\}$. The parity bits were punctured alternatively

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to generate a rate 1/2 turbo code. The length of the interleaver was $2^{15} = 32\,768$. Turbo code with these parameters has been evaluated on a variety of channels and the performance of CPM scheme coded with this turbo code can be easily compared to that of the other coded modulation schemes.

A CPM signaling can be specified by three parameters: number of data symbols M, the frequency response pulse $g(t)$ and the modulation index h. We tested the performance of CPM channels for $M = 2, 4$ and 8. In this paper, we provide results only for $M = 4$ and 8. It is well known that for $M = 2$, CPM is an inferior modulation scheme when compared to the CPM for $M = 4$ and 8 [4, page 182]. Our simulation results affirm this fact for turbo coded CPM channels also.

For full response signaling, we simulated continuous phase frequency shift keying (CPFSK) which is a special case of CPM modulation with rectangular frequency pulses. For partial response signaling, we used raised cosine frequency pulses of duration $3T$ (also referred to as 3RC-CPM signaling), where T is the symbol period [4, page 52]. We expect other full and partial response CPM signaling schemes to follow the similar trends as shown in this paper.

The choice of modulation index $h = K/P$ is more involved. For a given M and $g(t)$, increasing h results in larger occupied bandwidth. However, increasing h (up to a certain limit) usually also results in a larger minimum distance between CPM signals and thus provides larger asymptotic coding gains. Traditionally, the value of modulation index h is chosen to give a good trade-off between bandwidth and asymptotic coding gain. Note that the trend of increased minimum distance with larger values of h does not hold for certain values of h referred to as weak modulation indices. Weak modulation indices have been tabulated for various CPM schemes [4], and are avoided in practice due to their low asymptotic gains.

For turbo coded CPM signaling, we cannot use the aforementioned trade-offs to choose the modulation index h since turbo codes work in a range of low SNR's for which asymptotic coding gains lose their significance. Specifically, since the turbo code used here has rate 1/2, we are interested in the values of modulation index that requires least SNR to achieve the uncoded bit error rate of 13%–14%. Moreover, we are also interested in the values of h that have low P, since the complexity of demodulator is directly proportional to P [2]. Note that theoretically rate 1/2 can be achieved on a binary-input AWGN with approximately 15% uncoded bit error rate and on a binary symmetric channel (BSC) with approximately 11% uncoded bit error rate. It turns out that by using turbo codes, one can usually achieve rate 1/2 on a channel with 13%–14% uncoded bit error rate which lies in between the maximum theoretical uncoded bit error rate possible on AWGN channel and BSC.

Therefore, in order to choose h, we simulated uncoded CPM for different values of h, and chose an h that achieved a good trade-off between the value of P and the SNR required to achieve a 13% bit error rate. For CPFSK, and for $M = 2, 4$ and 8, good values of h turned out to be 0.7, 0.9, and 0.9 respectively. For 3RC-CPM, and for $M = 2, 4$ and 8, these values were 0.8, 1.0, and 1.0 respectively. Note that for 3RC-CPM scheme, we have chosen weak modulation indices for $M = 4$ and 8.

Figures 2–3 compare the simulated performance of turbo coded CPM signaling with that of turbo coded M-ary PSK. For $M = 4$, the Figure 2 shows SNR versus bit error rate curve for four turbo coded signaling schemes: PSK signaling, 3RC-CPM with $h = 1.0$, and CPFSK with $h = 0.9$ and 1.0. Clearly, compared to the PSK signaling, to achieve a bit error rate of 10^{-5}, 3RC-CPM with $h = 1.0$ and CPFSK with $h = 0.9$ respectively require approximately 1.4 dB and 1.6 dB more SNR. We note that the power spectrum of CPM signaling is more compact than that of the PSK signaling. Thus the extra SNR spent in CPM signaling buys a more compact power spectrum.

In Figure 2, we also present SNR versus bit error rate curve for CPFSK with $h = 1.0$. There are two reasons to compare the performances of CPFSK signaling with $h = 0.9$ and $h = 1.0$. First, for $h = 1.0$, the CPFSK demodulator is memoryless, resulting in a much simpler implementation, and second, $h = 1.0$ is a weak modulation index for CPFSK close to $h = 0.9$. Figure 2 shows that compared to CPFSK with $h = 0.9$, CPFSK with $h = 1.0$ requires about 0.7 dB more SNR to achieve a bit error rate of 10^{-5}.

Figure 3 shows the same performance curves for $M = 8$. Clearly, for $M = 8$, CPM signaling schemes operate within a few tenths of a dB away from the PSK signaling. Specifically, 3RC-CPM signaling is almost as energy efficient as the PSK signalling. Thus, CPM signaling provides a bandwidth efficient alternative to PSK signaling. The price paid for for the increase bandwidth efficiency is the higher complexity of the CPM demodulator with respect to a PSK demodulator.

4. Discussion

In this paper, we propose the use of bit interleaved coded modulation to convert M-ary CPM channels into binary-input channels and to apply turbo codes to CPM

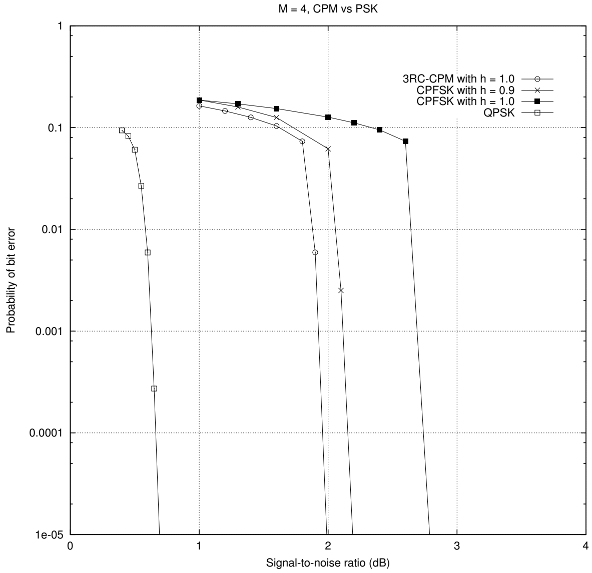

Figure 2: Turbo coded CPM versus turbo coded MPSK for $M = 4$

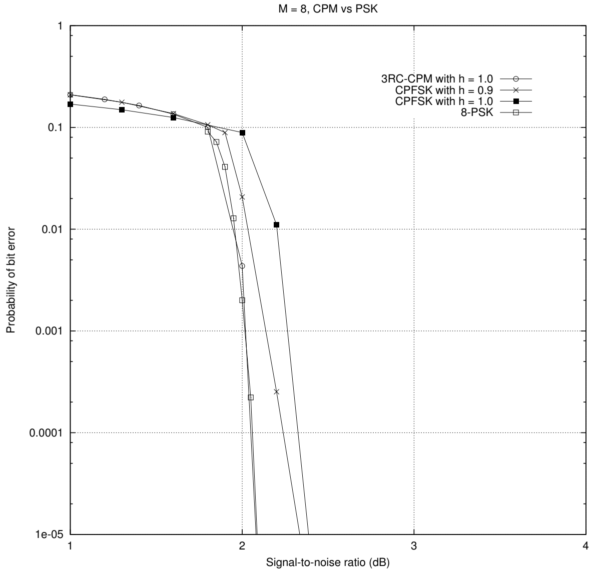

Figure 3: Turbo coded CPM versus turbo coded MPSK for $M = 8$

channels. We show that the CPM signaling provides a bandwidth-efficient alternative to the PSK signaling. For $M = 4$, there is a 1.4 dB–1.6 dB penalty in using CPM signaling, however for $M = 8$, CPM signaling is as energy efficient as PSK signaling.

Usually, weak modulation indices are avoided in practice due to the low asymptotic coding gain of the underlying CPM signaling. Our simulations show that when the coded systems operate at low SNR's, weak modulation indices do not carry significant penalties, and are in fact worth consideration due to the potential reduction in demodulator complexity.

Note that the proposed architecture uses two interleavers, one for the turbo encoder and another for bit interleaving. We strongly believe that it may be possible to eliminate one of the interleavers (thereby reducing the implementation complexity) and still be able to obtain the coding gains demonstrated here. This is a topic of current research.

Bit interleaved coded modulation (BICM) is inherently suboptimum in the sense that it reduces the information capacity of the communication channel. Our preliminary calculation of the capacity of CPM modulated AWGN channel shows that the resultant suboptimality is not significant, and BICM is a good design choice for applying binary turbo codes to the CPM modulated channels.

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