

# MAGNETIC RECORDING SYSTEM DESIGN TO REDUCE MEDIUM NOISE THROUGH SIGNAL PRECOMPENSATION

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## Abstract

Much of the noise in magnetic recording systems is due to intrinsic properties of the magnetic medium itself. Much of this noise is repeatable in that identical waveforms recorded in the same place on the medium (with an erasure in between) have highly correlated noise. This effect is exploited in the present paper through the design of systems to estimate and subsequently correct the distortion of the recorded waveform due to medium noise. The approach may be applicable to other storage channels whose noise is medium dependent such as magneto-optic media. A method for optimally reducing repeatable additive medium noise is proposed. Simulations of this system have been run and the results are promising.

## 1. Introduction

Most models of magnetic recording channels use a traditional communication theory model. The channel is the magnetic medium. The medium may corrupt the signal, and if modeled as additive noise, average statistics of this medium noise may be estimated and used in system design to improve the performance of the system. Other sources of noise include receiver noise and head noise. Even when these latter two noise sources are considered, the medium noise limits the system performance. The authors have developed models for magnetic media [1,2] as have others [3,4,5,6,7]. These models account for the fact that medium noise arises from the microscopic properties of the recording medium. These properties are deterministic once the medium is manufactured. While it is infeasible to have a complete microscopic scan of the medium for use in signal design, it may be possible to measure local medium features on-line and to use those features when designing the recorded signal. This strategy accounts not only for the average effects of the medium, but also for local effects.

The proposed strategy is to make on-line measurements of the medium noise and then to use these measurements in signal design. Possible ways to accomplish this strategy are discussed below. They may be classified as "write-read-write" recording strategies. First a diagnostic signal is written on the medium, second the resulting magnetization pattern is read, and third an information carrying signal is recorded. The design of the second written signal depends on the model for the medium noise.

There is no generally accepted model for the medium noise. There is some evidence that the medium noise has a large multiplicative portion. This does not adequately account for medium noise when the medium is demagnetized, however. Some initial simulations using our medium model indicate greater variability of the written magnetization in the demagnetized case than when a strong unidirectional magnetic field has been applied. This seems to be consistent with measurements from magnetic media. It should be noted that the noise in the demagnetized case is correlated with the previous signal written; however, presently we are not including previously written signals when modeling the magnetization resulting from writing a new signal. While our medium model has been useful for computing the capacity of magnetic media, it has not yet been used for designing magnetic recording systems. Presently our medium model is being used to analyze medium noise in received voltage waveforms [8].

The approach here is based on simplified models for the effects of medium noise. As we gain experience with this approach and as more sophisticated models become available, we will use them.

## 2. Write-Read-Write Precompensation: Additive Noise Case

We begin the analysis for this scheme with a linear approximation to the recording process. Recognizing that the process is inherently nonlinear, we anticipate that the approach presented here must be improved by using a more accurate nonlinear model for the recording system. Due to the speed of electronics, the approach proposed holds the possibility of being implemented in silicon.

Shown in Figure 1 is a block diagram description of the write-read-write recording process. Figure 1 shows three heads flying over a magnetic medium. The first records a diagnostic signal  $s_1(t)$ . The second reads the resulting magnetization on the medium. Third, a signal that has been computed by the electronics and includes the desired signal and compensation for the medium noise is written onto the medium. When the information is read at a later time, the signal-to-noise ratio will be significantly higher, resulting in better recovery of the desired signal. The crucial design of the electronics block is based on a model for the manner in which the medium noise is manifested. The discussion in this paper is based on an additive model for the medium noise.

Shown in Figure 2 is an approximate linear model of a magnetic recording system. In this system, all blocks are assumed to be linear and time-invariant, all random processes are assumed to be wide-sense stationary. A diagnostic signal  $s_1(t)$  is written using the write head  $h(t)$ , medium noise is added ( $n_d(t) + n_1(t)$ ), the signal is read ( $g(t)$ ) introducing electronics noise ( $w_1(t)$ ). There may be an equalizer in the system and this is incorporated into  $h(t)$  or  $g(t)$  as appropriate. The desired channel response is  $d(t)$ . We assume here that  $b(t) = (g * h)(t)$  (where  $*$  denotes convolution). Denoting the output of  $g(t)$  plus  $w_1(t)$  by  $y_1(t)$ , the error signal  $e(t) = y_1(t) - (b * s_1)(t)$  equals the part of the received voltage waveform due to system noise. The noise has a repeatable component,  $n_d(t)$ , due to the medium, and two unrepeatable components,  $n_1(t)$  due to the medium, and  $w_1(t)$  due to the electronics. The goal is to compensate for the repeatable component. To do this,  $e(t)$  is filtered, subtracted from  $s(t)$ , the information-bearing signal to be recorded, then recorded with the write head again. Physically, a signal is written on the medium then read; the desired signal,  $d(t)$ , computed electronically, is subtracted from this signal producing the error signal  $e(t)$ ; then the new signal  $(s(t) - (c * e)(t))$  is written at the same place on the medium.

The design problem may be stated as minimizing in the waveform  $y(t)$  that is eventually read the distortion due to the repeatable component of medium noise. From this viewpoint,  $c(t)$  may be designed from the reduced system shown in Figure 3. In that figure, the information-bearing signal  $s(t)$  and the diagnostic signal  $s_1(t)$  have been removed. Let the distortion be measured in terms of noise power. Then the goal is to design  $c(t)$  to minimize the signal power in  $y_e(t)$ , the noise component of the output. For any real system design, there will be a constraint class,  $\mathbf{C}$ , for the  $c(t)$  so that they are realizable. A typical constraint is that it be the output of a transversal filter with a fixed number of taps. The problem statement becomes

$$\min_{c \in \mathbf{C}} E[|(g * n_d)(t) - (g * h * c * e)(t)|^2]. \quad (1)$$

For many constraints, the problem reduces to solving the well-known normal equations.

To minimize (1), substitute

$$e(t) = y_1(t) - (b * s_1)(t) = w_1(t) + (g * (n_1 + n_d))(t). \quad (2)$$

Computing the expected value, the problem statement (2) reduces to

$$\min_{c \in \mathbf{C}} [c * \tilde{c} * q(0) - 2(\tilde{c} * p)(0)], \quad (3)$$

where for any signal  $f(t)$ ,  $\tilde{f}(t) = f(-t)$ ,

$$q(t) = (b * \tilde{b} * (R_{ww} + g * \tilde{g} * (R_{dd} + R_{nn}))) (t), \quad (4)$$

$$p(t) = (g * \tilde{g} * \tilde{b} * R_{dd})(t), \quad (5)$$

$$b(t) = (g * h)(t),$$

and  $R_{ww}(t)$ ,  $R_{nn}(t)$ , and  $R_{dd}(t)$  are the autocovariance functions for  $w_i(t)$ ,  $n_i(t)$ , and  $n_d(t)$ , respectively ( $i = 1$  or  $2$ ).

The unconstrained solution to (3), if it exists is

$$C_u(\omega) = P(\omega)/Q(\omega), \quad (6)$$

where  $P(\omega)$  and  $Q(\omega)$  are the Fourier transforms of  $p(t)$  and  $q(t)$ , respectively,  $C_u(\omega)$  is the Fourier transform of  $c_u(t)$ , and the subscript  $u$  in  $c_u(t)$  represents the fact that this is an unconstrained optimum solution. This may not exist because  $Q(\omega)$  may have zeros. Also, even if this solution exists, it may not be realizable. Usually some type of constraint is added so that the solution for  $c(t)$  exists and is well-behaved (easily realizable). In the following section, the constraint of the solution being implementable by a tapped delay line (and thus easily realizable using standard VLSI design) is added.

Notice that if all noise were additive and Gaussian, and the unconstrained solution (6) is used, then the limiting increase in capacity is determined by the change in the power spectrum of the channel. As opposed to using a standard writing scheme, the component due to the repeatable medium noise is attenuated. This attenuation increases as the energy of the repeatable component increases relative to the energy of the unrepeatable component. That is, if the power spectrum due to the noise in an uncompensated system is  $S_y(\omega) = S_r(\omega) + S_u(\omega)$ , where  $S_r(\omega)$  is the repeatable component, then the power spectrum of the compensated system is

$$S_{y_e}(\omega) = S_u(\omega) + S_r(\omega) \frac{S_u(\omega)}{S_r(\omega) + S_u(\omega)}. \quad (7)$$

Note that  $S_u(\omega) = |G(\omega)|^2 S_{nn}(\omega) + S_{ww}(\omega)$  and  $S_r(\omega) = |G(\omega)|^2 S_{dd}(\omega)$ ; here  $S_{ww}$ ,  $S_{nn}$ , and  $S_{dd}$  are the Fourier transforms of  $R_{ww}$ ,  $R_{nn}$ , and  $R_{dd}$ , respectively. The capacity of the Gaussian channel is then increased; the increase depends on the spectral shapes (that is, system factors such as the  $g(t)$ ,  $h(t)$ , and the noise levels) and detailed analysis of the standard water-filling capacity formula [9, p. 267]. In our experimental setup, depending on the medium tested, we have found that the repeatable component is one half to nine tenths the total noise power. If all of the repeatable medium noise is adequately modeled as additive noise, equation (7) implies a potential increase in signal to noise ratio of 1.2 to 7.2 dB.

### 3. Tapped Delay Line Implementation

As mentioned in the previous section, the actual implementation would be different from (6). Some realizability constraint must be imposed. One natural constraint (but not the only possible one; other constraints have been considered and the following derivation can easily be modified to account for other constraints) is that the implementation be realized by a tapped delay line. Tapped delay lines (finite impulse response filters with a finite number of coefficients) may be easily built using VLSI technol-

ogy.

The constraint class is the set of  $c(t)$  such that

$$c(t) = \sum_{n=-N}^N c[n]\delta(t - nT), \quad (8)$$

where  $\delta$  is a Dirac delta function, and  $T$  is the time interval between the taps. For each integer  $n$ , let

$$p[n] = p(nT) \quad \text{and} \quad q[n] = q(nT). \quad (9)$$

The solution for the optimal tap weights is obtained by substituting the form (8) for  $c(t)$  into (3), then taking the derivatives with respect to the tap weights  $c[n]$ . This results in a set of  $2N + 1$  equations in the  $2N + 1$  unknown tap weights  $c[n]$ . The equations are

$$\sum_k q[n-k]c[k] = p[n], \quad \text{for } -N \leq n \leq N. \quad (10)$$

If the values of  $c[n]$  and  $p[n]$  are put into vectors  $\mathbf{c}$  and  $\mathbf{p}$ , and the values of  $q[n]$  are put into the matrix  $\mathbf{Q}$  such that the  $n, k$  entry of  $\mathbf{Q}$  is  $q[n-k]$ , then equation (10) may be rewritten compactly as

$$\mathbf{Q}\mathbf{c} = \mathbf{p}. \quad (11)$$

This equation may be solved directly to give

$$\mathbf{c}_{opt} = \mathbf{Q}^{-1}\mathbf{p}. \quad (12)$$

Note that in many cases of interest,  $q$ ,  $c$ , and  $p$  may have symmetry properties. From (4), and using the fact that autocovariance functions are even,  $q(t)$  is an even function of  $t$ . This implies that  $\mathbf{Q}$  is symmetric,  $\mathbf{Q}^T = \mathbf{Q}$ . In addition,  $\mathbf{Q}$  is a Toeplitz matrix, a fact that may be exploited in reducing the computation in (12). In many cases of interest,  $g(t)$  is an odd function of  $t$  (as in the Lorentz pulse as presented in the next section). If  $h(t)$  is an even function of  $t$ , then  $p(t)$  is an odd function of  $t$ . These symmetry properties may be stated compactly as  $q[n] = q[-n]$  and  $p[n] = -p[-n]$ . If  $\mathbf{Q}$  is nonsingular, these imply that the solution for  $c$  is odd symmetric so  $c[n] = -c[-n]$ . Thus there are only  $N$  values in  $\mathbf{c}$  that must be found, and the matrix that must be inverted is only  $N \times N$  rather than  $(2N + 1) \times (2N + 1)$  as in (12). If  $\mathbf{Q}$  is singular, then a minimum squared error solution of (11) subject to the symmetry constraint may be computed.

#### 4. Simulations

In this first simulation we made many simplifying assumptions. We assume that the impulse response function  $h(t)$  of the write head is  $\delta(t)$  and the unit step response function of the read head is a Lorentz pulse. We also assume that the head mechanism is precise enough to let us write at a predecided position on the medium.

We assume that all noise sources are white and Gaussian. Additive repeatable media noise, head noise, and electronic noise are assumed to have total noise power distributed among themselves in the ratio of 8:1:1. Total

noise power is 10% of the the signal power.

Let  $\frac{1}{T_p}$  be the flux reversal rate for saturation magnetic recording (this is the nominal bit rate). The step response of the read head is assumed to be given by

$$\frac{1}{1 + \left(\frac{2t}{T_p S}\right)^2}$$

Here  $T_p S$  is the half width of the Lorentz pulse, where  $S$  determines the width of the Lorentz pulse relative to  $T_p$ . Differentiating this with respect to  $t$  we get the impulse response function  $g(t)$  of the read head

$$g(t) = \frac{-\left(\frac{8t}{T_p^2 S^2}\right)}{\left[1 + \left(\frac{2t}{T_p S}\right)^2\right]^2}.$$

The Fourier transform of  $g(t)$  equals

$$G(f) = j\pi^2 T_p S f \exp(-\pi T_p S |f|).$$

Let the sampling rate for the tapped delay line be  $f_s = 1/T = K/T_p$ . Here  $K$  is the number of times a potential flux reversal is sampled. Then the energy in the frequencies greater than  $f_s/2$  gets aliased and the total aliased energy is

$$E_a(f_s) = 2 \int_{\frac{f_s}{2}}^{\infty} |G(f)|^2 df \\ = v \exp(-X) \left[1 + X + \frac{X^2}{2}\right]$$

where  $v = \pi/(2T_p S)$  and  $X = \pi SK$ . From the graph of  $E(X)$  versus  $X$ , it can be concluded that for  $X = 12$  less than 0.1% energy gets aliased.

In our simulation we choose  $S = 1/\pi$ , which represents a fairly high linear density and we choose  $K = 10$ , at which less than 0.3% of the energy of  $g(t)$  and less than 4% of the energy of a square wave get aliased. This percentage of aliased energy is high for  $K = 10$ , however it does not matter in this simulation since by exploiting linearity of the system, we can compute noise powers independently of the signal. Note that  $g[n]$ , the discretized version of  $g(t)$ , is nonzero even for large  $|n|$ . However it is monotonically decreasing in magnitude. For our simulation we have truncated  $g[n]$  suitably in such a manner that the truncated  $g[n]$ ,  $(-8 \leq n \leq 8)$  has more than 99% of the energy of the original nontruncated  $g[n]$ . Tap weights in the filter  $c[n]$  were computed assuming  $-16 \leq n \leq 16$ .

Simulation results show that under the assumptions stated above, the write-read-write scheme yields 4.74 dB reduction in noise power on the average. Figure 4 com-

compares the noise powers before and after signal precompensation obtained in 1000 runs. Figures 5 and 6 show typical read waveforms obtained by writing the signal without and with compensation.

## 5. Conclusions

Recently, experiments have demonstrated that medium noise has a repeatable component [10]. In this paper we have outlined a new strategy for precompensating for local medium effects that are additive through the use of a write-read-write recording protocol. Equations have been derived for the design of the optimal compensation filter based on the statistics of the noise sources. The performance of this filter in an ideal situation has been predicted, and simulations have demonstrated the improved performance.

We have also explored the use of more sophisticated models for the effects of medium noise, including multiplicative effects. Initial studies show that for multiplicative noise, the relatively simple analysis presented above is not adequate. While estimates of the effects of the multiplicative and additive components may be estimated, their optimal use in signal compensation is complicated.

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