# How Do Networks Become Navigable?

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# Motivation

**Observation: real networks**

- *have small diameter* $O(\log n)$
- *have high clustering* $c_{real} \gg c_{rand}$
- *are (often) navigable using only 'local' information*

**The Small World (Stanley Milgram)**

- *6-degrees of separation study*
- *illustrated navigability of social network*

**WWW Analogy**

- *without search engines, web is still navigable*

# Motivation

**Question**

- *How do real networks become (locally) navigable?*

**Idea**

- *By distributed modifications to the topology that improve ‘local navigability’*
- *e.g. WWW surfers changing links on their home pages based on their navigation of the network*

# Main Results

- *Network navigability emerges from a decentralized and topologically dynamic network model.*
- *Links are modified according to the results of searches performed on the network.*
- *The link-length ($\ell$) distribution converges to a powerlaw of the form* $f(\ell) \sim \ell^{-\alpha_{\text{rewired}}}$.
- $\alpha_{\text{rewired}}$ *similar to theoretical results (Kleinberg)*
- *Rewired routing times are* $O(\log^2 n)$ *and match theoretical optimum*

# Details

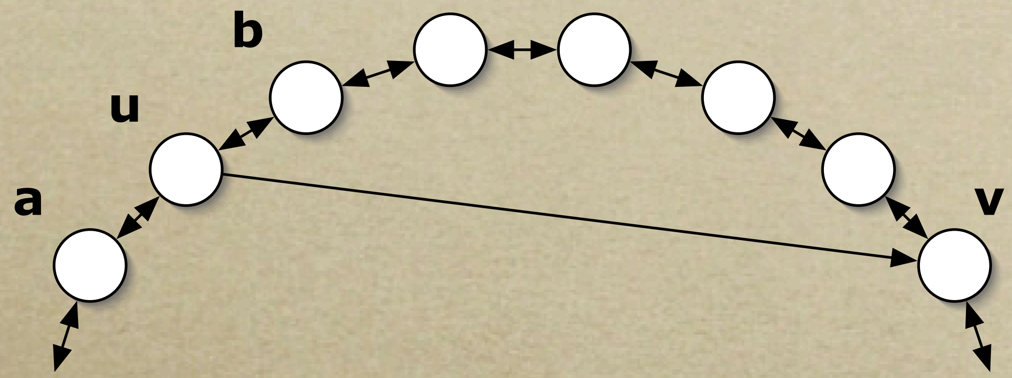
**Random graph models**

- *short paths exist,*
- *but are hard to find*

**Concepts of 'distance'**

- *lattice distance vs.*
- *topological distance*

# Network Model - Structure




1. *d-dimensional lattice (d=1)*
2. *(fixed) local neighborhood radius (p=1)*
3. *(free) long range links (q=1)*
4. *Manhattan distance metric* $\ell = |u - v|$
5. *greedy routing algorithm*

# Network Model - Dynamics

1. *choose source-destination pair* $(x, y)$ *uniformly at random*
2. *choose 'expected length'* $T_{\mathrm{thresh}}$ *for journey uniformly random* $1 \leq T_{\mathrm{thresh}} \leq |x - y|$
3. *after* $T_{\mathrm{thresh}}$ *topological hops, the surfer becomes 'frustrated'*
4. *frustrated surfers 'bookmark' the node at which they became frustrated, changing source's long range link*

# Simulation Parameters

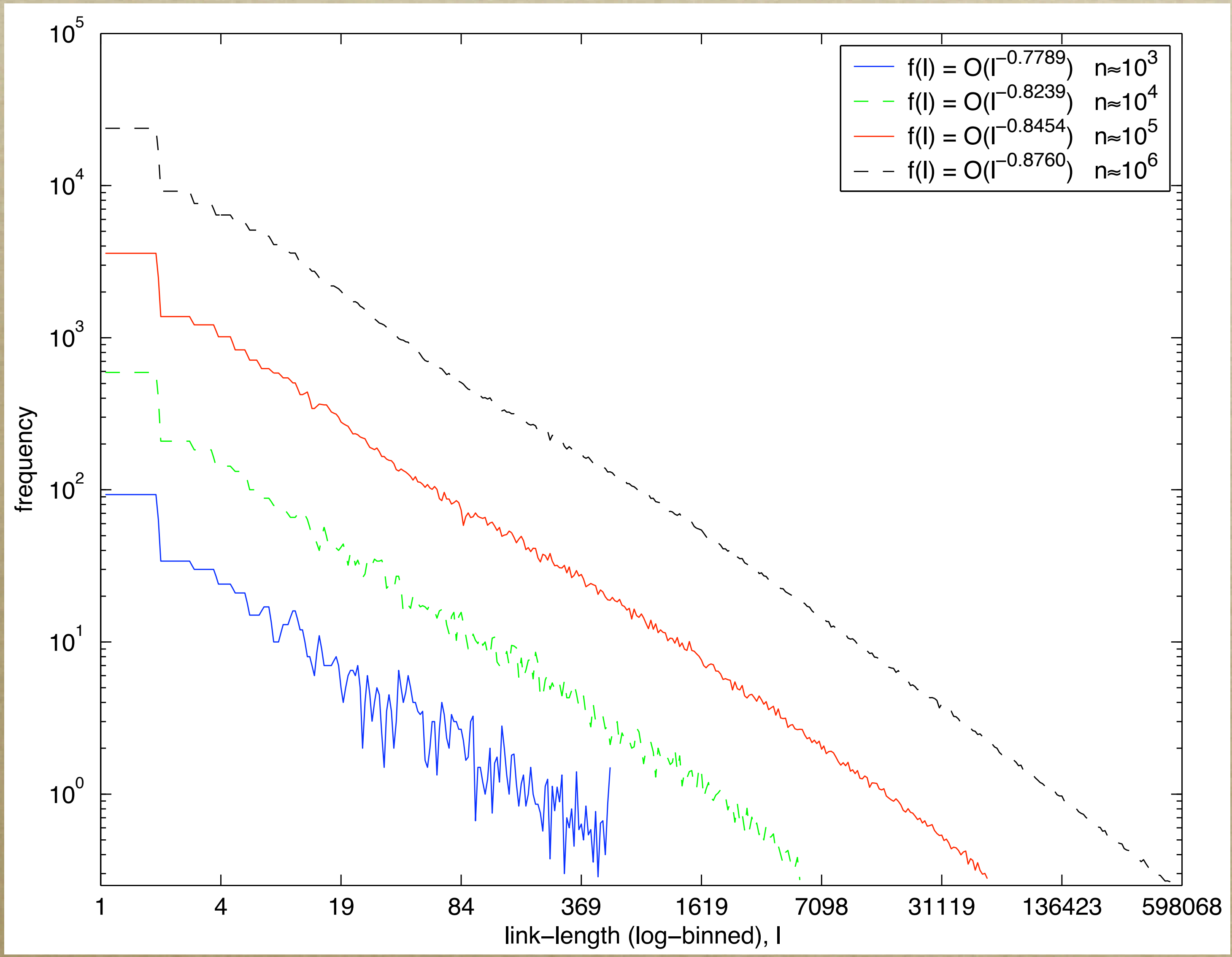
**Unless otherwise noted**

- *'Long-range' links are initially self-loops.*
- *Rewiring process was run until link-length distribution became stable.*

# Rewired Link-Lengths

- *Powerlaw distribution emerges from rewiring process*
- *Slope is similar to Kleinberg's theoretical optimum*

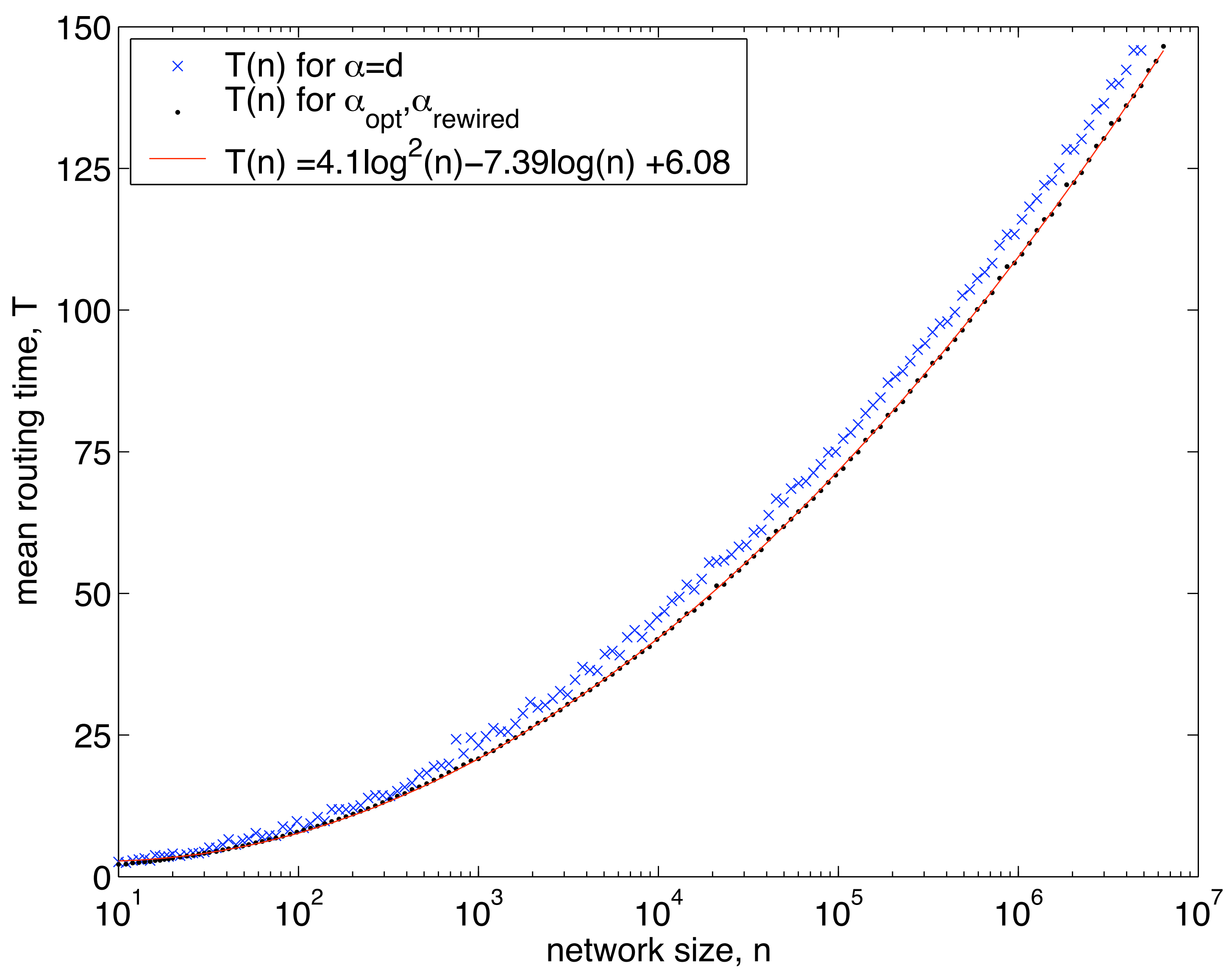
# Rewired Link-Lengths



# Rewired Routing Times

- *Rewired graphs achieve fast* $O(\log^2 n)$ *routing times*
- *Kleinberg's 'optimum' static graphs achieve same routing times*

# Rewired Routing Times



# Time to Rewire

- *The 'rewiring time'* $\tau(n)$ *is defined as the number of rewiring trials per node until*

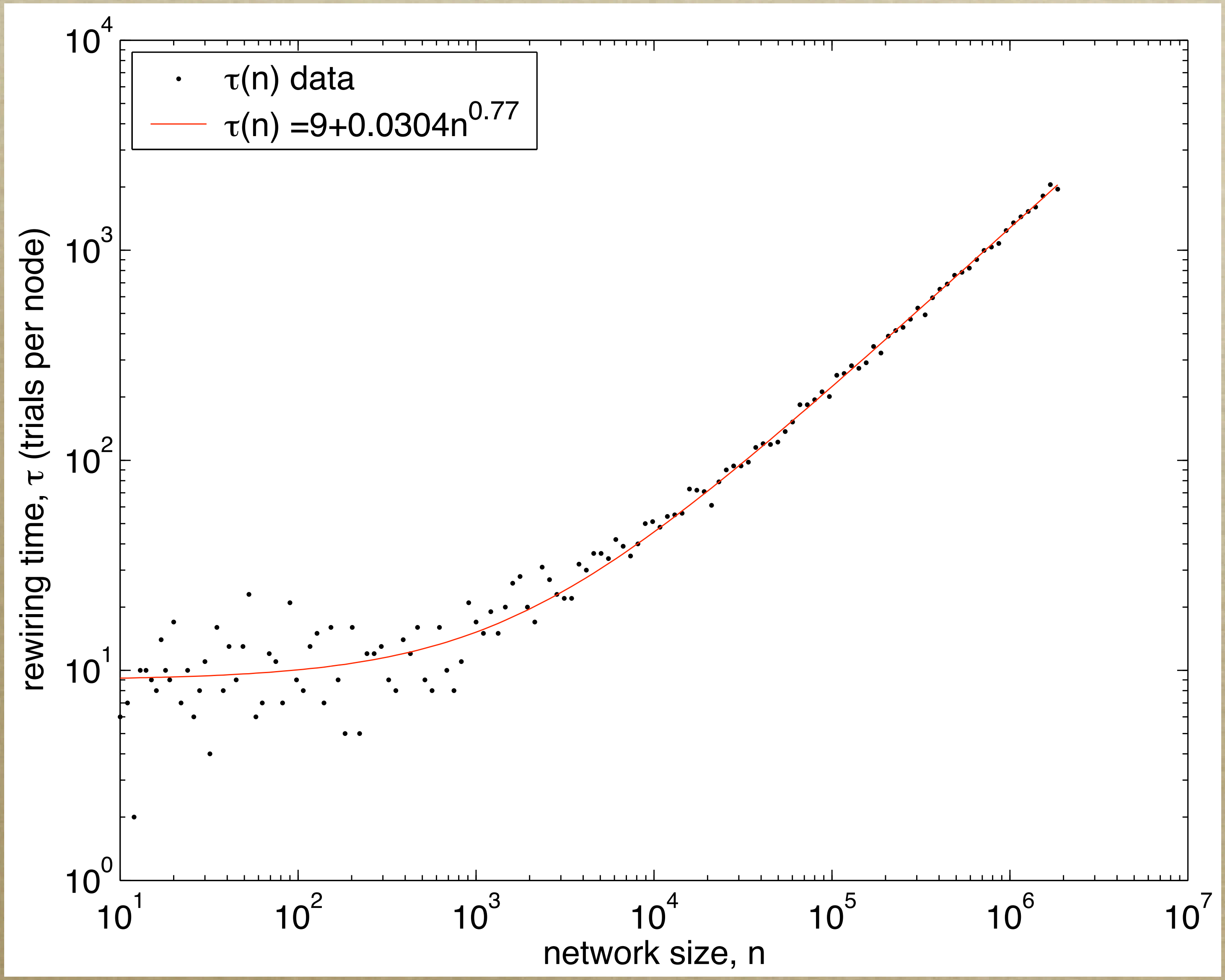
$$T_{\text{rewired}} \leq 1.01 \cdot T_{\text{K}}$$

*and grows as a low-order polynomial*

- $$\tau(n) \sim n^{0.77}$$

*for large* $n$

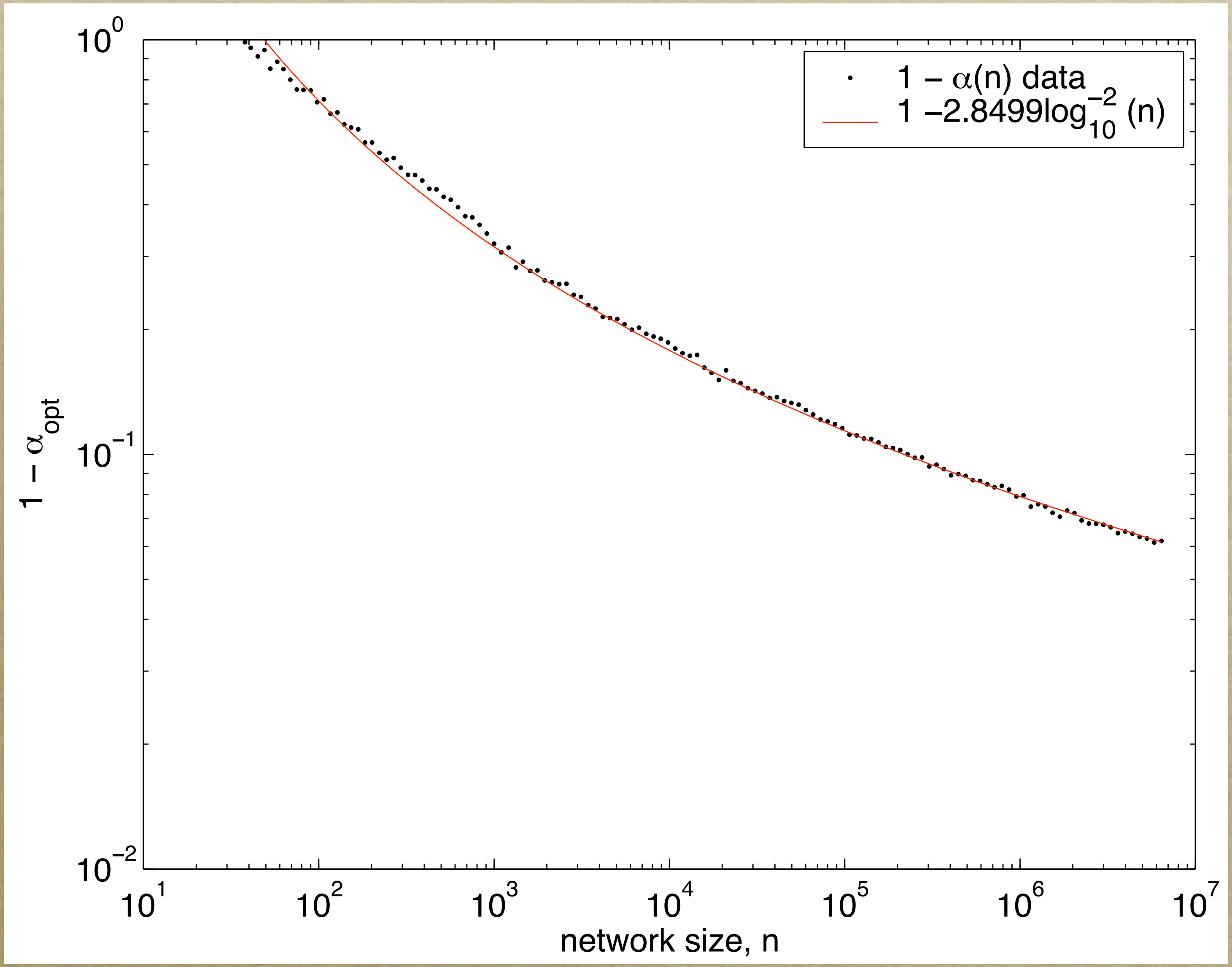
# Time to Rewire



# Finite-Size Effects

- *Finite-size effects cause optimal routing times on Kleinberg static graphs to deviate from theory:* $\alpha_{\mathrm{K}} \neq d$
- *Empirically, we see that* $\alpha_{\mathrm{K}}(n) = 1 - \dfrac{A}{\log^2 n}$
- *And* $\alpha_{\text{rewired}} \sim \alpha_{\mathrm{K}}$
- *So, we should expect* $\lim_{n \to \infty} \alpha_{\text{rewired}} = d$

# Finite-Size Effects



# Conclusions

- *Distributed rewiring process achieves fast routing times* $O(\log^2 n)$ *and builds powerlaw link-length distributions*
- *Natural interpretation: users of the network modify their local topology to improve success of similar future searches*
- *Adaptive process: ignorant of network size, performance and topology.*
- *Applications: p2p networks (i.e. Freenet), sensor networks, etc.*

# New Directions

- *Explore possibility of better rewiring algorithms*
- *Explore other 'demand distributions,' both random and non-random*
- *Apply rewiring process to Kleinberg and Watts/Dodds/ Newman tree-structured social network*
- *Apply rewiring process to a real system: Freenet*
- *Analytically understand why rewiring process works; physics or mathematical proofs*

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- *Preprint is available at*
  http://www.arxiv.org/abs/cond-mat/0309415

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